

5.4 e^x Integration

Pg. 352 #'s 91-115 odd, 63, 66

91) $\int e^{5x} (5) dx$ $u=5x$
 $du=5dx$
 $\int e^u du$
 $e^{5x} + C$

93) $\int e^{2x-1} du$ $u=2x-1$
 $du=2dx$
 $\frac{1}{2} \int e^u du$
 $\frac{1}{2} e^{2x-1} + C$

95) $\int x^2 e^{x^3} dx$ $u=x^3$
 $du=3x^2 dx$
 $\frac{1}{3} \int e^u du$
 $\frac{1}{3} e^{x^3} + C$

97) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ $u=x^{1/2}$
 $du=\frac{1}{2} x^{-1/2} dx$
 $2 \int e^u du$
 $2e^{\sqrt{x}} + C$

99) $\int \frac{e^{-x}}{1+e^{-x}} dx$ $u=1+e^{-x}$
 $du=-e^{-x} dx$
 $-\int \frac{1}{u} du$
 $-\ln|1+e^{-x}| + C$

101) $\int e^x \sqrt{1-e^x} dx$ $u=1-e^x$
 $du=-e^x dx$
 $-\int u^{1/2} du$
 $-\frac{2}{3} (1-e^x)^{3/2} + C$

103) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$ $u=e^x - e^{-x}$
 $du=(e^x + e^{-x}) dx$
 $\int \frac{1}{u} du$
 $\ln|e^x - e^{-x}| + C$

105) $\int \frac{5-e^x}{e^{2x}} dx$
 $5 \int e^{-2x} dx - \int e^{-x} dx$
 $u=-2x$ $v=-x$
 $du=-2dx$ $dv=-dx$
 $-\frac{1}{2} du = dx$

107) $\int e^{-x} \tan(e^{-x}) dx$ $u=e^{-x}$
 $du=-e^{-x} dx$
 $-\int \tan(u) du$
 $-\ln|\cos(e^{-x})| + C$

109) $\int_0^1 e^{-2x} dx$ $u=-2x$
 $du=-2dx$
 $-\frac{1}{2} \int_0^1 e^u du$
 $u(1)=-2$
 $u(0)=0$

$-\frac{5}{2} \int e^u du + \int e^v dv$
 $-\frac{5}{2} e^{-2x} + e^{-x} + C$

111) $\int_0^1 x e^{-x^2} dx$ $u=-x^2$
 $du=-2x dx$
 $-\frac{1}{2} \int_0^1 e^u du$
 $u(1)=-1$
 $u(0)=0$

$\frac{1}{2} \int_{-2}^0 e^u du$
 $\frac{1}{2} e^u \Big|_{-2}^0 = \frac{1}{2} e^0 - \frac{1}{2} e^{-2} = \frac{1}{2} - \frac{1}{2e^2}$

$\frac{1}{2} e^u \Big|_{-1}^0 = \frac{1}{2} e^0 - \frac{1}{2} e^{-1}$
 $\frac{1}{2} - \frac{1}{2e}$

$$113) \int_1^3 \frac{e^{3/x}}{x^2} dx \quad u = 3/x$$

$$du = -3/x^2 dx$$

$$-\frac{1}{3} \int_3^1 e^u du \quad -\frac{1}{3} du = \frac{1}{x^2} dx$$

$$\frac{1}{3} \int_1^3 e^u du \quad u(3) = 1$$

$$\frac{1}{3} e^u \Big|_1^3 = \frac{1}{3} e^3 - \frac{1}{3} e^1$$

$$115) \int_0^3 \frac{2e^{2x}}{1+e^{2x}} dx \quad u = 1+e^{2x}$$

$$du = 2e^{2x} dx$$

$$\int_2^{1+e^6} \frac{1}{u} du \quad u(3) = 1+e^6$$

$$u(0) = 1+1 = 2$$

$$\ln|u| \Big|_2^{1+e^6} = \ln|1+e^6| - \ln|2|$$

$$63) xe^y - 10x + 3y = 0$$

$$xe^y \frac{dy}{dx} + e^y(1) - 10 + 3 \frac{dy}{dx} = 0$$

$$xe^y \frac{dy}{dx} + 3 \frac{dy}{dx} = 10 - e^y$$

$$\frac{dy}{dx} (xe^y + 3) = 10 - e^y$$

$$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$$

$$66) 1 + \ln xy = e^{x-y}$$

$$0 + \frac{1}{xy} (x \frac{dy}{dx} + y(1)) = e^{x-y} (1 - \frac{dy}{dx})$$

$$\frac{1}{y} \frac{dy}{dx} + \frac{1}{x} = e^{x-y} - e^{x-y} \frac{dy}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} + e^{x-y} \frac{dy}{dx} = e^{x-y} - \frac{1}{x}$$

$$\frac{dy}{dx} \left(\frac{1}{y} + e^{x-y} \right) = e^{x-y} - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{e^{x-y} - \frac{1}{x}}{\frac{1}{y} + e^{x-y}}$$

$$\frac{dy}{dx} = \frac{e^0 - \frac{1}{1}}{\frac{1}{1} + e^0} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

$$\boxed{y=1}$$