

Calculus Section 5.4 e^x Integration

-Integrate the natural exponential function

Homework: page 352 #'s 91-113 odd, 108, 63, 66

Integration Rules for Exponential Functions

Let u be a differentiable function of x .

$$1) \int e^x dx = e^x + C$$

$$2) \int e^u du = e^u + C$$

Examples)

$$1) \int e^{3x+1} dx$$

$$u = 3x+1$$

$$du = 3dx$$

$$\frac{1}{3} \int e^u du$$

$$\frac{1}{3} du = dx$$

$$\boxed{\frac{1}{3} e^{3x+1} + C}$$

$$2) \int 5xe^{-x^2} dx$$

$$u = -x^2$$

$$du = -2x dx$$

$$-\frac{5}{2} \int e^u du$$

$$-\frac{1}{2} du = x dx$$

$$\boxed{-\frac{5}{2} e^{-x^2} + C}$$

$$3) \int \frac{e^{1/x}}{x^2} dx$$

$$u = 1/x$$

$$du = -\frac{1}{x^2} dx$$

$$-\int e^u du$$

$$-du = \frac{1}{x^2} dx$$

$$\boxed{-e^{1/x} + C}$$

$$4) \int \sin x \cdot e^{\cos x} dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-\int e^u du$$

$$-du = \sin x dx$$

$$\boxed{-e^{\cos x} + C}$$

$$5) \int_0^1 \frac{e^x}{1+e^x} dx$$

$$u = 1 + e^x$$

$$du = e^x dx$$

$$u(1) = 1 + e$$

$$u(0) = 1 + e^0 = 2$$

$$\int_2^{1+e} \frac{1}{u} du$$

$$\ln|u| \Big|_2^{1+e}$$

$$\ln|1+e| - \ln|2|$$

$$\boxed{\ln\left(\frac{1+e}{2}\right)}$$

$$6) \int \frac{2+e^x}{e^{3x}} dx$$

$$2 \int e^{-3x} dx + \int e^{-2x} dx$$

$$u = -3x$$

$$du = -3dx$$

$$-\frac{1}{3} du = dx$$

$$v = -2x$$

$$dv = -2dx$$

$$-\frac{1}{2} dv = dx$$

$$-\frac{2}{3} \int e^u du - \frac{1}{2} \int e^v dv$$

$$\boxed{-\frac{2}{3} e^{-3x} - \frac{1}{2} e^{-2x} + C}$$