Calculus Section 5.4 e^x Properties and Derivative

-Develop properties of the natural exponential function
-Differentiate natural exponential functions

Homework: page 352 #'s 1, 3, 5, 11, 13, 15, 33-47 odd, 67, 83

Definition of the Natural Exponential Function (e^x)
The function f(x) = ln(x) has an inverse because it passes the horizontal line test. The inverse of f(x) = ln(x) is the exponential function $g(x) = e^x$. Because ln(x) and e^x are inverses, it holds that: $\ln(e^x) = x$ and $e^{\ln(x)} = x$.

Solving Exponential and Logarithmic Equations

1) $7 = e^{x+1}$

$\ln(7) = \ln(e^{x+1})$

$\ln 7 = x+1$

$\boxed{x = \ln 7 - 1}$

2) $\ln(2x-3) = 5$

$e^{\ln(2x-3)} = e^5$

$2x-3 = e^5$

$2x = e^5 + 3$

$\boxed{x = \frac{e^5+3}{2}}$

Review of Exponent Rules

1) $e^a e^b = e^{a+b}$

2) $\frac{e^a}{e^b} = e^{a-b}$

Examples)

$e^4(e^2) =$ e^6

$\frac{e^5}{e^2} =$ e^3

$\frac{e}{e^5} =$ $e^{-4} = \frac{1}{e^4}$

$e^0 =$ 1

Properties of the Natural Exponential Function

1) Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

2) e^x is continuous and increasing on its domain.

3) e^x is concave up on its entire domain.

4) $\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to\infty} e^x = \infty$

Derivative of the Natural Exponential Function
Let u be a differentiable function of x.

1) $\frac{d}{dx}[e^x] = e^x$ 2) $\frac{d}{dx}[e^u] = e^u \cdot du$

Examples)

1) $\frac{d}{dx}[e^{2x-1}]$

$e^{2x-1} \cdot (2)$

$\boxed{2e^{2x-1}}$

2) $\frac{d}{dx}[e^{-3/x}]$

$e^{-\frac{3}{x}} \cdot \left(\frac{3}{x^2}\right)$

$\boxed{\frac{3e^{-3/x}}{x^2}}$

$\frac{d}{dx}[-3x^{-1}] = 3x^{-2}$

3) $\frac{d}{dx}[x^2 e^{4x}]$

$x^2(e^{4x} \cdot 4) + e^{4x}(2x)$

$\boxed{4x^2e^{4x} + 2xe^{4x}}$

Example)
The projected population y (in thousands) of California from 2015 through 2030 can be modeled by $y = 34{,}696e^{0.0097t}$ where t represents the year, with t = 15 corresponding to 2015. At what rate will the population be changing in 2020?

$y' = 34696e^{.0097t} \cdot (.0097)$

$y' = 336.5512e^{.0097t}$

$y'(30) = 336.5512e^{.0097(30)}$

$\boxed{y'(30) = 450.226 \text{ thousand people/year}}$

Example)
Determine $\frac{dy}{dx}$ for the function $y^2 = 2e^{xy}$

$2y\frac{dy}{dx} = 2e^{xy} \cdot \left(x\frac{dy}{dx} + y\right)$

$2y\frac{dy}{dx} = 2xe^{xy}\frac{dy}{dx} + 2ye^{xy}$

$2y\frac{dy}{dx} - 2xe^{xy}\frac{dy}{dx} = 2ye^{xy}$

$\frac{dy}{dx}(2y - 2xe^{xy}) = 2ye^{xy}$

$\boxed{\frac{dy}{dx} = \frac{2ye^{xy}}{2y - 2xe^{xy}}}$