Calculus Section 5.5 Bases Other Than e

-Differentiate exponential/log functions that have bases other than e
-Integrate exponential/log functions that have bases other than e

Homework: page 362 #'s 37-47 odd, 59, 71-81 odd

Derivatives for Bases Other Than e
Let *a* be a positive real number ($a \neq 1$) and let *u* be a differentiable function of x.

1) $\frac{d}{dx}[a^u] = (\ln a)a^u \, du$

2) $\frac{d}{dx}[\log_a u] = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot du$

Proof:

$a^u = e^{\ln a^u} = e^{u \ln a}$

$\frac{d}{dx}\left[e^{u \ln a}\right] = e^{u \ln a} \cdot (\ln a) \cdot du$

$e^{\ln a^u} \cdot (\ln a) = a^u(\ln a) \cdot du$

Proof:

$\log_a u = \frac{\ln u}{\ln a} = \frac{1}{\ln a} \ln u$

$\frac{d}{dx}\left[\frac{1}{\ln a} \ln u\right] = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot du$

Examples)

1) $\frac{d}{dx}[5^x]$

$\boxed{(\ln 5)5^x}$

2) $\frac{d}{dx}[2^{3x}]$

$(\ln 2)2^{3x} \cdot (3)$

$\boxed{(3\ln 2)2^{3x}}$

3) Write the equation for the line tangent to $y = x^2(4^x)$ at the place where $x = 1$.

$y' = x^2\left((\ln 4)4^x\right) + 4^x(2x)$

$y'(1) = (1)^2(\ln 4)4^1 + 4^1(2)$

$y'(1) = 4\ln 4 + 8$

$y(1) = (1^2)(4^1)$

$y(1) = 4$

$\boxed{y - 4 = (4\ln 4 + 8)(x - 1)}$

4) Find the 2nd derivative of $f(x) = 3^x$

$f'(x) = (\ln 3)3^x$

$f''(x) = (\ln 3)(\ln 3)3^x = \boxed{(\ln 3)^2 3^x}$

5) $y = \log_{10}\cos x$

$$y' = \frac{1}{\ln 10} \cdot \frac{1}{\cos x} \cdot (-\sin x)$$

$$\boxed{y' = \frac{-\sin x}{(\ln 10)\cos x}}$$

6) $y = \log_5 3x$

$$y' = \frac{1}{\ln 5} \cdot \frac{1}{3x} \cdot (3)$$

$$\boxed{y' = \frac{3}{(\ln 5)(3x)}}$$

Integrals for Bases Other Than e

Let *a* be a positive real number (a ≠ 1) and let *u* be a differentiable function of x.

$$\int a^u du = \frac{1}{\ln a} a^u + C$$

Proof: $a^u = e^{\ln a^u} = e^{u \ln a}$

$$\int e^{(\ln a)u} du \qquad v = (\ln a)u$$

$$dv = (\ln a)\, du$$

$$\frac{1}{\ln a}\int e^v dv \qquad \frac{1}{\ln a} dv = du$$

$$\frac{1}{\ln a} e^v \rightarrow \frac{1}{\ln a} e^{(\ln a)u} \longrightarrow \frac{1}{\ln a} a^u$$

Examples

1) $\int 2^x dx$

$$\boxed{\frac{1}{\ln 2} 2^x + C}$$

2) $\int x(5^{-x^2})dx$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$-\frac{1}{2}\int 5^u du$$

$$-\frac{1}{2} \cdot \frac{1}{\ln 5} 5^{-x^2} + C$$

$$\boxed{\frac{-5^{-x^2}}{2\ln 5} + C}$$

3) $\int_{-1}^{2} 3^x dx$

$$\frac{1}{\ln 3} 3^x \Big|_{-1}^{2}$$

$$\frac{1}{\ln 3} 3^2 - \frac{1}{\ln 3} 3^{-1}$$

$$\frac{9}{\ln 3} - \frac{1}{3\ln 3}$$

$$\boxed{\frac{26}{3\ln 3}}$$