Section 5.5 Exponential Functions with Function Bases

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63) $y = x^{2/x}$

$\ln y = \frac{2}{x}\ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x}\cdot\frac{1}{x} + \ln x\left(-2x^{-2}\right)$

$\frac{dy}{dx} = \left(\frac{2}{x^2} - \frac{2\ln x}{x^2}\right)y$

$\frac{dy}{dx} = \frac{2-2\ln x}{x^2}\left(x^{2/x}\right)$

64) $y = x^{x-1}$

$\ln y = (x-1)\ln x$

$\frac{1}{y}\frac{dy}{dx} = (x-1)\frac{1}{x} + \ln x\,(1)$

$\frac{dy}{dx} = \left(\frac{x-1}{x} + \ln x\right)x^{x-1}$

67) $y = x^{\sin x}$ $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\ln y = (\sin x)\ln x$

$\frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \ln x(\cos x)$

$\frac{dy}{dx} = \left[\frac{\sin x}{x} + \ln x(\cos x)\right]x^{\sin x}$

@ $\frac{\pi}{2}$

$\frac{dy}{dx}\left(\frac{\pi}{2}\right) = \left[\frac{\sin\frac{\pi}{2}}{\pi/2} + \ln\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2}\right]\frac{\pi}{2}^{\sin\pi/2}$

$= \left[\frac{2}{\pi} + 0\right]\frac{\pi}{2}$

$= 1$

$y - \frac{\pi}{2} = 1\left(x - \frac{\pi}{2}\right)$

68) $y = (\sin x)^{2x}$ $\left(\frac{\pi}{2}, 1\right)$

$\ln y = 2x\ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = 2x\frac{1}{\sin x}\cos x + \ln(\sin x)(2)$

$\frac{dy}{dx} = \left[\frac{2x}{\sin x}\cos x + 2\ln(\sin x)\right](\sin x)^{2x}$

@ $\pi/2$

$\frac{dy}{dx} = \left[\frac{\pi}{1}(0) + 2\ln(1)\right](1)^{\pi}$

$\frac{dy}{dx} = [0 + 0](1) = 0$

$y - 1 = 0\left(x - \frac{\pi}{2}\right)$

$y = 1$

69) $y = (\ln x)^{\cos x}$ $(e, 1)$

$\ln y = \cos x \ln(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{1}{\ln x}\cdot\frac{1}{x} + \ln(\ln x)(-\sin x)$

$\frac{dy}{dx} = \left[\frac{\cos x}{x \ln x} - \sin x \ln(\ln x)\right](\ln x)^{\cos x}$

@ $x = e$

$\frac{dy}{dx} = \left[\frac{\cos(e)}{e \ln e} - \sin(e)\ln(\ln e)\right](\ln e)^{\cos(e)}$

$\frac{dy}{dx} = \left[\frac{\cos(e)}{e} - \sin(e)\ln(1)\right](1)^{\cos(e)}$

$\frac{dy}{dx} = \left[\frac{\cos(e)}{e} - 0\right](1) = \frac{\cos(e)}{e}$

$y - 1 = \frac{\cos(e)}{e}(x - e)$

70) $y = x^{1/x}$ $(1, 1)$

$\ln y = \frac{1}{x}\ln(x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\left(\frac{1}{x}\right) + (\ln x)\left(\frac{-1}{x^2}\right)$

$\frac{dy}{dx} = \left[\frac{1}{x^2} - \frac{1}{x^2}\ln x\right]x^{1/x}$

@ $x = 1$

$\frac{dy}{dx}(1) = \left[\frac{1}{1} - \frac{1}{1}\ln(1)\right](1)^1$

$\frac{dy}{dx}(1) = [1 - 0](1) = 1$

$y - 1 = 1(x - 1)$