5.6 Inverse Trig Differentiation

Pg. 372 #'s 39-48, 77, 78

39) $f(x) = 2\arcsin(x-1)$

$f'(x) = 2\left(\frac{1}{\sqrt{1-(x-1)^2}}\right)$

$\boxed{f'(x) = \frac{2}{\sqrt{1-(x-1)^2}}}$

40) $f(t) = \arcsin(t^2)$

$\boxed{f'(t) = \frac{2t}{\sqrt{1-t^4}}}$

41) $g(x) = 3\arccos\left(\frac{x}{2}\right)$

$g'(x) = 3\left(\frac{-1/2}{\sqrt{1-(x/2)^2}}\right)$

$\boxed{g'(x) = \frac{-3}{2\sqrt{1-(x/2)^2}}}$

42) $f(x) = \operatorname{arcsec}(2x)$

$f'(x) = \frac{2}{|2x|\sqrt{(2x)^2-1}}$

$\boxed{f'(x) = \frac{2}{|2x|\sqrt{4x^2-1}}}$

43) $f(x) = \arctan e^x$

$f'(x) = \frac{e^x}{(e^x)^2+1}$

$\boxed{f'(x) = \frac{e^x}{e^{2x}+1}}$

44) $f(x) = \arctan\sqrt{x}$

$f'(x) = \frac{\frac{1}{2}x^{-1/2}}{(\sqrt{x})^2+1}$

$f'(x) = \frac{1}{2\sqrt{x}(x+1)}$

$\boxed{f'(x) = \frac{1}{2x^{3/2}+2x^{1/2}}}$

45) $g(x) = \frac{\arcsin 3x}{x}$

$g'(x) = \frac{x\left(\frac{3}{\sqrt{1-9x^2}}\right) - \arcsin 3x\,(1)}{x^2}$

$\boxed{g'(x) = \frac{\frac{3x}{\sqrt{1-9x^2}} - \arcsin 3x}{x^2}}$

46) $h(x) = x^2\arctan 5x$

$h'(x) = x^2\left(\frac{5}{(5x)^2+1}\right) + \arctan 5x\,(2x)$

$\boxed{h'(x) = \frac{5x^2}{25x^2+1} + 2x\arctan 5x}$

47) $h(t) = \sin(\arccos t)$

$h'(t) = \cos(\arccos t)\left(\frac{-1}{\sqrt{1-t^2}}\right)$

$h'(t) = \frac{-\cos(\arccos t)}{\sqrt{1-t^2}}$

$\boxed{h'(t) = \frac{t}{\sqrt{1-t^2}}}$

48) $f(x) = \arcsin x + \arccos x$

$f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}}$

$\boxed{f'(x) = 0}$

77) $x^2 + x\arctan y = y - 1 \quad \left(\frac{-\pi}{4}, 1\right)$

$2x + x\left(\frac{\frac{dy}{dx}}{y^2+1}\right) + \arctan y(1) = \frac{dy}{dx}$

$\frac{dy}{dx}\left(\frac{x}{y^2+1}\right) - \frac{dy}{dx} = -2x - \arctan y$

$\frac{dy}{dx}\left(\frac{x}{y^2+1} - 1\right) = -(2x + \arctan y)$

$\frac{dy}{dx} = \frac{-(2x + \arctan y)}{\frac{x}{y^2+1} - 1}$

@ $\left(\frac{-\pi}{4}, 1\right)$

$\frac{dy}{dx} = \frac{-\left(\frac{-\pi}{2} + \arctan(1)\right)}{\frac{-\pi/4}{1+1} - 1}$

$\frac{dy}{dx} = \frac{-\left(\frac{-\pi}{2} + \frac{\pi}{4}\right)}{-\frac{\pi}{8} - 1}$

$\frac{dy}{dx} = \frac{-4\pi + 2\pi}{\pi + 8} = \frac{-2\pi}{\pi+8}$

$\boxed{y - 1 = \frac{-2\pi}{\pi+8}\left(x + \frac{\pi}{4}\right)}$

78) $\arctan(xy) = \arcsin(x+y) \quad (0,0)$

$\frac{x\frac{dy}{dx} + y(1)}{(xy)^2 + 1} = \frac{1 + \frac{dy}{dx}}{\sqrt{1-(x+y)^2}}$

@ $(0,0)$

$\frac{0\frac{dy}{dx} + 0}{(0)^2 + 1} = \frac{1 + \frac{dy}{dx}}{\sqrt{1-(0)^2}}$

$0 = \frac{1 + \frac{dy}{dx}}{\sqrt{1}}$

$0 = 1 + \frac{dy}{dx}$

$\frac{dy}{dx} = -1$

$y - 0 = -1(x - 0)$

$\boxed{y = -x}$