Calculus Section 5.6 Inverse Trig Differentiation

-Differentiate an inverse trig function.

Homework: page 372 #'s 39 – 48, 77, 78

Derivatives of Inverse Trig Functions

Let u be a differentiable function of x.

1) $\frac{d}{dx}[\arcsin u] = \frac{du}{\sqrt{1-u^2}}$

2) $\frac{d}{dx}[\arccos u] = \frac{-du}{\sqrt{1-u^2}}$

3) $\frac{d}{dx}[\arctan u] = \frac{du}{u^2+1}$

4) $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-du}{u^2+1}$

5) $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{du}{|u|\sqrt{u^2-1}}$

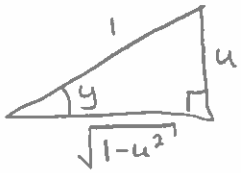
6) $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-du}{|u|\sqrt{u^2-1}}$

Proof)

$y = \arcsin(u)$

$\sin y = \sin(\arcsin(u))$

$\sin y = u$



$\frac{d}{dx}[\sin y = u]$

$\cos y \frac{dy}{dx} = du$

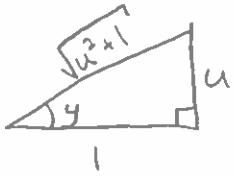
$\frac{dy}{dx} = \frac{du}{\cos y}$

$\frac{dy}{dx} = \frac{du}{\sqrt{1-u^2}}$

$y = \arctan(u)$

$\tan y = \tan(\arctan(u))$

$\tan y = u$



$\frac{d}{dx}[\tan y = u]$

$\sec^2 y \frac{dy}{dx} = du$

$\frac{dy}{dx} = (\cos^2 y)du$

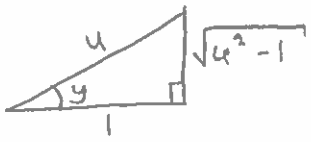
$\frac{dy}{dx} = \left(\frac{1}{\sqrt{u^2+1}}\right)^2 du$

$\frac{dy}{dx} = \frac{du}{u^2+1}$

$y = \operatorname{arcsec}(u)$

$\sec y = \sec(\operatorname{arcsec}(u))$

$\sec y = u$



$\frac{d}{dx}[\sec y = u]$

$\sec y \tan y \frac{dy}{dx} = du$

$\frac{dy}{dx} = \cos y \cot y \, du$

$\frac{dy}{dx} = \left(\frac{1}{u}\right)\left(\frac{1}{\sqrt{u^2-1}}\right) du$

$\frac{dy}{dx} = \frac{du}{|u|\sqrt{u^2-1}}$

abs. because hypotenuse must be positive

Examples)

1) $\frac{d}{dx}[\arcsin(2x)]$

$\frac{2}{\sqrt{1-(2x)^2}}$

$\boxed{\frac{2}{\sqrt{1-4x^2}}}$

2) $\frac{d}{dx}[\arctan(3x)]$

$\frac{3}{(3x)^2+1}$

$\boxed{\frac{3}{9x^2+1}}$

3) $\frac{d}{dx}[\arccos(\sqrt{x})]$

$$\frac{-\frac{1}{2}x^{-1/2}}{\sqrt{1-(\sqrt{x})^2}}$$

$$\boxed{\frac{-1}{2\sqrt{x}\sqrt{1-x}}}$$

4) $\frac{d}{dx}[\operatorname{arcsec}(e^{2x})]$

$$\frac{2e^{2x}}{|e^{2x}|\sqrt{(e^{2x})^2-1}}$$

$$\frac{2e^{2x}}{e^{2x}\sqrt{e^{4x}-1}}$$

$$\boxed{\frac{2}{\sqrt{e^{4x}-1}}}$$

Where are the points of inflection for the graph of $y = (\arctan x)^2$?

$$y = (\arctan x)^2$$

$$y' = 2(\arctan x)^1\left(\frac{1}{x^2+1}\right)$$

$$y' = \frac{2\arctan x}{x^2+1}$$

$$y'' = \frac{(x^2+1)\left(\frac{2}{x^2+1}\right) - 2\arctan x\,(2x)}{(x^2+1)^2}$$

$$y'' = \frac{2-4x\arctan x}{(x^2+1)^2}$$

$$0 = \frac{2-4x\arctan x}{(x^2+1)^2}$$

$$0 = 2-4x\arctan x$$

graph in calculator

$$\boxed{x = \pm 0.765}$$