Calculus Section 5.6 Inverse Trig Functions

-Develop properties of the six inverse trigonometric functions

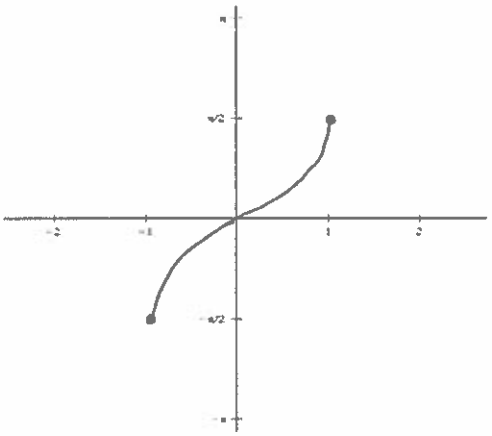
Homework: page 372 #'s 3–13 odd, 15-20, 25–35 odd (Hint #35 take sin of both sides)

None of the six trig functions have inverses. Being periodic makes all of them fail to horizontal line test.

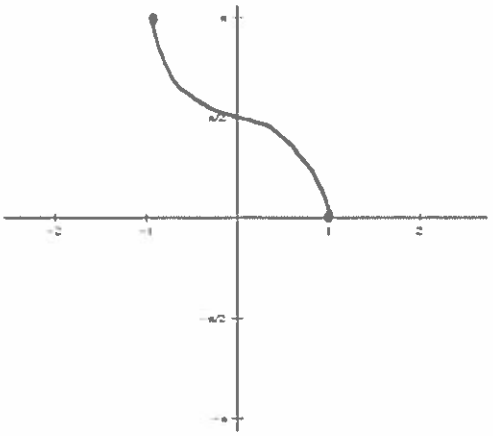
However, you can restrict the domain for the trig functions to allow them to have an inverse.

Inverse Trig Functions and Their Restricted Domains

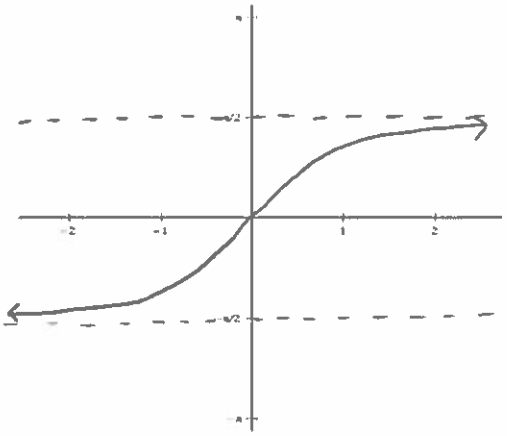
☆ arcsinx D: $[-1,1]$



arccosx D: $[-1,1]$



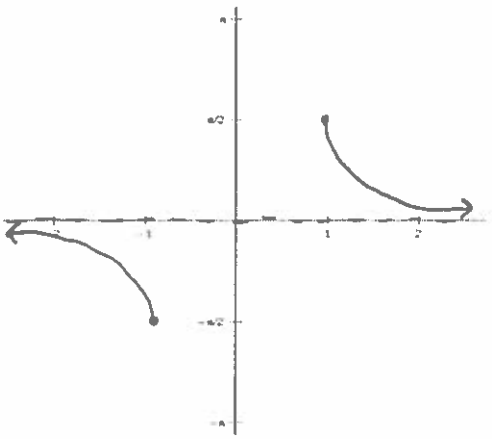
☆☆☆ arctanx D: $(-\infty,\infty)$



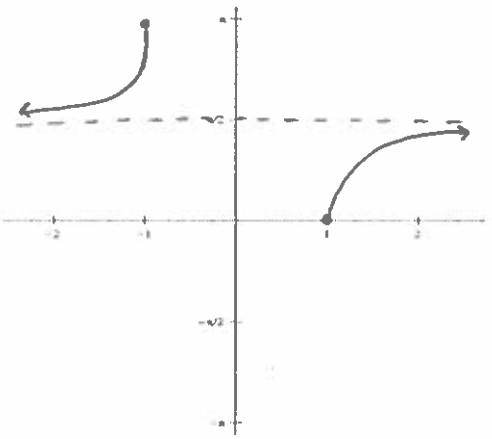
$\lim_{x\to\infty} \arctan x = \pi/2$

$\lim_{x\to-\infty} \arctan x = -\pi/2$

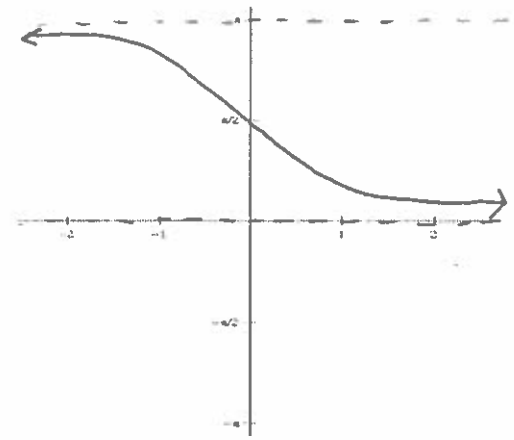
arccscx D: $(-\infty,-1]\cup[1,\infty)$



☆ arcsecx D: $(-\infty,-1]\cup[1,\infty)$



arccotx D: $(-\infty,\infty)$



Examples

1) arcsin(-1/2)

$y = \arcsin(-1/2)$

$\sin y = \sin(\arcsin(-1/2))$

$\sin y = -1/2$

$\boxed{y = -\pi/6}$

2) arccos(0)

$y = \arccos(0)$

$\cos y = \cos(\arccos(0))$

$\cos y = 0$

$\boxed{y = \pi/2}$

3) arctan($\sqrt{3}$)

$y = \arctan(\sqrt{3})$

$\tan y = \tan(\arctan(\sqrt{3}))$

$\tan y = \sqrt{3}$

$\boxed{y = \frac{\pi}{3}}$

$\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\sqrt{3}/2}{1/2}$

4) arctan(2x – 3) = $\frac{\pi}{4}$

$\tan(\arctan(2x-3)) = \tan\frac{\pi}{4}$

$2x - 3 = 1$

$2x = 4$

$\boxed{x = 2}$

5) Use a calculator to evaluate arccsc(2.590)

$y = \operatorname{arccsc}(2.590)$

$\csc y = \csc(\operatorname{arccsc}(2.590))$

$\frac{1}{\sin y} = 2.590$

$\sin y = \frac{1}{2.590}$

$y = \arcsin(\frac{1}{2.590})$

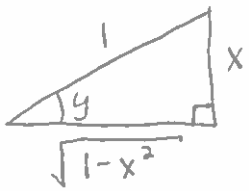
$\boxed{y = .396}$

Using Right Triangles

4) Find cos(arcsin(x))

$y = \arcsin x$

$\sin y = x$



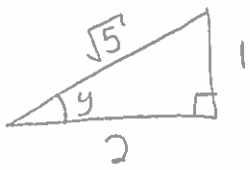
$\cos y = \sqrt{1-x^2}$

$\boxed{\cos(\arcsin x) = \sqrt{1-x^2}}$

5) Find tan $\left(arcsec\left(\frac{\sqrt{5}}{2}\right)\right)$

$y = \operatorname{arcsec}\frac{\sqrt{5}}{2}$

$\sec y = \frac{\sqrt{5}}{2}$



$\tan y = \frac{1}{2}$

$\boxed{\tan(\operatorname{arcsec}(\frac{\sqrt{5}}{2})) = \frac{1}{2}}$