5.7 Inverse Trig Integration

Pg. 380 #'s 1-8, 33

1) $\int \frac{dx}{\sqrt{9-x^2}}$ $a=3$ $u=x$ $du=dx$

$\int \frac{du}{\sqrt{a^2-u^2}}$

$\arcsin\frac{u}{a}+C$

$\arcsin\frac{x}{3}+C$

2) $\int \frac{dx}{\sqrt{1-4x^2}}$ $a=1$ $u=2x$ $du=2dx$ $\frac{1}{2}du=dx$

$\frac{1}{2}\int \frac{du}{\sqrt{a^2-u^2}}$

$\frac{1}{2}\arcsin\frac{u}{a}+C$

$\frac{1}{2}\arcsin 2x+C$

3) $\int \frac{1}{x\sqrt{4x^2-1}}dx$ $a=1$ $u=2x \rightarrow x=\frac{1}{2}u$ $du=2dx$ $\frac{1}{2}du=dx$

$\frac{1}{2}\int \frac{du}{\frac{1}{2}u\sqrt{a^2-a^2}}$

$2\cdot\frac{1}{2}\int \frac{du}{u\sqrt{u^2-a^2}}$

$\frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+C$

$\operatorname{arcsec}|2x|+C$

4) $\int \frac{12}{1+9x^2}dx$ $a=1$ $u=3x$ $du=3dx$ $\frac{1}{3}du=dx$

$12\cdot\frac{1}{3}\int \frac{du}{a^2+u^2}$

$4\cdot\frac{1}{a}\arctan\frac{u}{a}+C$

$4\arctan 3x+C$

5) $\int \frac{1}{\sqrt{1-(x+1)^2}}dx$ $u=x+1$ $a=1$ $du=dx$

$\int \frac{du}{\sqrt{a^2-u^2}}$

$\arcsin\frac{u}{a}+C$

$\arcsin(x+1)+C$

6) $\int \frac{1}{4+(x-3)^2}dx$ $u=x-3$ $a=2$ $du=dx$

$\int \frac{du}{a^2+u^2}$

$\frac{1}{a}\arctan\frac{u}{a}+C$

$\frac{1}{2}\arctan\left(\frac{x-3}{2}\right)+C$

7) $\int \frac{t}{\sqrt{1-t^4}}\,dt$ $u=t^2$ $a=1$

$du=2t\,dt$

$\frac{1}{2}du=dt$

$\frac{1}{2}\int \frac{du}{\sqrt{a^2-u^2}}$

$\frac{1}{2}\arcsin\frac{u}{a}+C$

$\boxed{\frac{1}{2}\arcsin t^2+C}$

8) $\int \frac{1}{x\sqrt{x^4-4}}\,dx$ $u=x^2$ $a=2$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$\frac{1}{2}\left(\frac{1}{x}\right)du=dx$

$\frac{1}{2}\int \frac{\frac{1}{x}du}{x\sqrt{u^2-a^2}}$

$\frac{1}{2}\int \frac{du}{x^2\sqrt{u^2-a^2}}$

$\frac{1}{2}\int \frac{du}{u\sqrt{u^2-a^2}}$

$\frac{1}{2}\cdot\frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+C$

$\boxed{\frac{1}{4}\operatorname{arcsec}\frac{x^2}{2}+C}$

33) $\int_0^2 \frac{dx}{x^2-2x+2}$

$\left(x^2-2x+\underline{(-1)^2}\right)+2-\underline{(-1)^2}$

$(x-1)^2+2-1$

$(x-1)^2+1$

$\int_0^2 \frac{dx}{(x-1)^2+1}$ $u=x-1$ $a=1$

$du=dx$

$u(2)=1$

$u(0)=-1$

$\int_{-1}^{1} \frac{du}{u^2+a^2}$

$\frac{1}{a}\arctan\frac{u}{a}\Big|_{-1}^{1}$

$\arctan u\Big|_{-1}^{1}$

$\arctan(1)-\arctan(-1)$

$\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)$

$\frac{2\pi}{4}=\boxed{\frac{\pi}{2}}$