**Euler's Method to Approximate the Solution to a Differential Equation** Name: Answer Key

1) Let y = f(x) be the particular solution to the differential equation y' = x + y with initial condition f(0) = 1. Use Euler's Method to approximate f(1) using 5 equal step sizes. $\Delta x = .2$

| x | 0 | .2 | .4 | .6 | .8 | 1 |
|---|---|---|---|---|---|---|
| y | 1 | 1.2 | 1.48 | 1.856 | 2.3472 | 2.97664 |

$\boxed{f(1) \approx 2.97664}$

$y_1 = y_0 + .2(0+1)$

$y_1 = 1 + .2(1)$

$y_1 = 1.2$

$y_2 = y_1 + .2(.2+1.2)$

$y_2 = 1.2 + .2(1.4)$

$y_2 = 1.48$

$y_3 = y_2 + .2(.4+1.48)$

$y_3 = 1.48 + .2(1.88)$

$y_3 = 1.856$

$y_4 = y_3 + .2(.6+1.856)$

$y_4 = 1.856 + .2(2.456)$

$y_4 = 2.3472$

$y_5 = y_4 + .2(.8+2.3472)$

$y_5 = 2.3472 + .2(3.1472)$

$y_5 = 2.97664$

2) Given $\frac{dy}{dx} = 3x - 2y$ and y(1) = 3, approximate y(0) using Euler's Method with 4 equal step sizes.

$\Delta x = -.25$

| x | 1 | .75 | .5 | .25 | 0 |
|---|---|---|---|---|---|
| y | 3 | 3.75 | 5.0625 | 7.21875 | 10.640625 |

$\boxed{y(0) \approx 10.640625}$

$y_1 = y_0 - .25(3(1) - 2(3))$

$y_1 = 3 - .25(-3)$

$y_1 = 3.75$

$y_2 = y_1 - .25(3(.75) - 2(3.75))$

$y_2 = 3.75 - .25(-5.25)$

$y_2 = 5.0625$

$y_3 = y_2 - .25(3(.5) - 2(5.0625))$

$y_3 = 5.0625 - .25(-8.625)$

$y_3 = 7.21875$

$y_4 = y_3 - .25(3(.25) - 2(7.21875))$

$y_4 = 7.21875 - .25(-13.6875)$

$y_4 = 10.640625$

3) Use Euler's Method with 4 equal step sizes to approximate the value of y(4) given y(2) = -2 and $\frac{dy}{dx} = y$.

$\Delta x = .5$

| x | 2 | 2.5 | 3 | 3.5 | 4 |
|---|---|---|---|---|---|
| y | -2 | -3 | -4.5 | -6.75 | -10.125 |

$\boxed{y(4) \approx -10.125}$

$y_1 = y_0 + .5(-2)$

$y_1 = -2 - 1$

$y_1 = -3$

$y_2 = y_1 + .5(-3)$

$y_2 = -3 - 1.5$

$y_2 = -4.5$

$y_3 = y_2 + .5(-4.5)$

$y_3 = -4.5 - 2.25$

$y_3 = -6.75$

$y_4 = y_3 + .5(-6.75)$

$y_4 = -6.75 - 3.375$

$y_4 = -10.125$

4) Given the differential equation $\frac{dy}{dx} = 0.5x(3 - y)$ and initial condition y(1) = 2. Use separation of variables to determine the value of y(0). Next, use Euler's Method with 2 equal step sizes to approximate y(0).

$\frac{dy}{dx} = .5x(3-y)$

$\int \frac{1}{3-y}\,dy = \int \frac{1}{2}x\,dx$

$u = 3-y$

$du = -dy$

$-du = dy$

$-\int \frac{1}{u}\,du = \frac{1}{4}x^2 + C$

$-\ln|3-y| = \frac{1}{4}x^2 + C$

@ (1,2)

$-\ln|3-2| = \frac{1}{4}(1)^2 + C$

$0 = \frac{1}{4} + C$

$-1/4 = C$

$-\ln|3-y| = \frac{1}{4}x^2 - \frac{1}{4}$

$\ln|3-y| = -\frac{1}{4}x^2 + \frac{1}{4}$

$e^{\ln|3-y|} = e^{-\frac{1}{4}x^2 + \frac{1}{4}}$

$3 - y = e^{-\frac{1}{4}x^2 + \frac{1}{4}}$

$y = 3 - e^{-\frac{1}{4}x^2 + \frac{1}{4}}$

$y(0) = 3 - e^{-\frac{1}{4}(0)^2 + \frac{1}{4}}$

$\boxed{y(0) = 1.716}$

$\Delta x = -.5$

| x | 1 | .5 | 0 |
|---|---|---|---|
| y | 2 | 1.75 | 1.59375 |

$y_1 = y_0 - .5(.5(1)(3-2))$

$y_1 = 2 - .5(.5)$

$y_1 = 1.75$

$y_2 = y_1 - .5(.5(.5)(3-1.75))$

$y_2 = 1.75 - .5(.3125)$

$y_2 = 1.59375$

$\boxed{y(0) \approx 1.59375}$