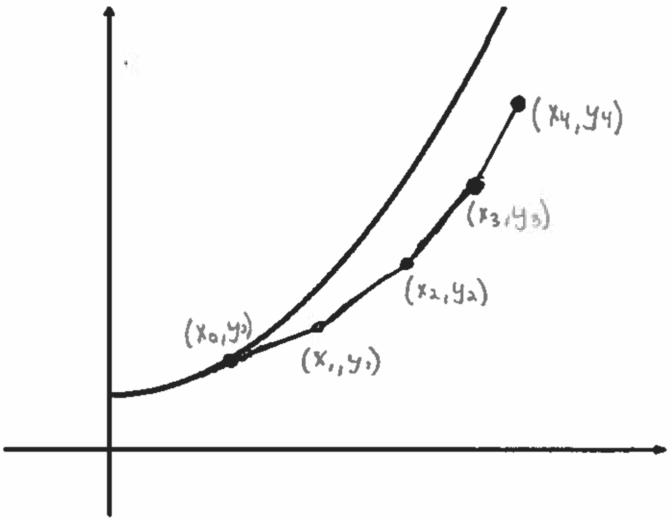
Calculus Section 6.1 Euler's Method to Approximate Diff Eqs

-Use Euler's Method to approximate solutions of differential equations

Homework: Euler's Method to Approximate the Solution to Differential Equations Worksheet

<u>Euler's Method</u> is a numerical approach to approximating the particular solution of a differential equation. Euler's Method uses the concept of <u>local linearity</u> to approximate the shape of the solution curve. Use the slope of the curve to create short, connected line segments.



-Use the slope at x_0 to create a tangent segment.
-The length of the segment is predetermined and called the <u>step-size</u> (h or Δx).
-The next segment (from x_1 to x_2) uses the slope at x_1 to find the next point x_2.
-This process continues until you reach the value that you are trying to find (i.e. approx. f(1.4)).

How to find the next point in the process:
Each seccessive point found using Euler's Method comes from the point slope formula.

$y - y_1 = m(x - x_1)$ point slope

$y_2 - y_1 = m(x_2 - x_1)$ (x_2, y_2) is next point

$\Delta x = x_2 - x_1$

$y_2 - y_1 = \Delta x(m)$

$y_2 = y_1 + \Delta x\, m$

$y_2 = y_1 + \Delta x\left(\frac{dy}{dx}\right)$ evaluate $\frac{dy}{dx}$ @ the point (x_1, y_1)

Next, $y_3 = y_2 + \Delta x\left(\frac{dy}{dx}\right)$ evaluate $\frac{dy}{dx}$ @ the point (x_2, y_2)

In general:
The smaller the step size, the better the approximation will be.

The approximation will be an overestimate if the curve is concave down and an underestimate if the curve is concave up.

Example)

Let y = f(x) be the particular solution to the differential equation y' = x – y with initial condition f(0) = 1. Use Euler's Method to approximate f(1.5) using 5 equal step sizes. $\Delta x = .3$

x	0	.3	.6	.9	1.2	1.5
y	1	.7	.58	.586	.6802	.83614

$\boxed{f(1.5) \approx .83614}$

$y_1 = y_0 + .3(0-1)$

$y_1 = 1 + .3(-1)$

$y_1 = .7$

$y_2 = y_1 + .3(.3 - .7)$

$y_2 = .7 + .3(-.4)$

$y_2 = .58$

$y_3 = y_2 + .3(.6 - .58)$

$y_3 = .58 + .3(.02)$

$y_3 = .586$

$y_4 = y_3 + .3(.9 - .586)$

$y_4 = .586 + .3(.314)$

$y_4 = .6802$

$y_5 = y_4 + .3(1.2 - .6802)$

$y_5 = .6802 + .3(.5198)$

$y_5 = .83614$

Example)

Given $\frac{dy}{dx} = 2x + y$ and y(1) = 3, approximate y(0) using Euler's Method with ~~5~~ 4 equal step sizes.

$\Delta x = -.25$

x	1	.75	.5	.25	0
y	3	1.75	.9375	.453125	.21484375

$\boxed{y(0) \approx .21484375}$

$y_1 = y_0 - .25(2(1) + 3)$

$y_1 = 3 - .25(5)$

$y_1 = 1.75$

$y_2 = y_1 - .25(2(.75) + 1.75)$

$y_2 = 1.75 - .25(3.25)$

$y_2 = .9375$

$y_3 = y_2 - .25(2(.5) + .9375)$

$y_3 = .9375 - .25(1.9375)$

$y_3 = .453125$

$y_4 = y_3 - .25(2(.25) + .453125)$

$y_4 = .453125 - .25(.953125)$

$y_4 = .21484375$