

6.3 Separation of Variables

Pg. 421 1, 3, 5, 7, 9, 15, 17, 23, 25

$$1) \frac{dy}{dx} = \frac{x}{y}$$

$$\int y dy = \int x dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$\boxed{y^2 = x^2 + C}$$

$$3) x^2 + 5y \frac{dy}{dx} = 0$$

$$5y \frac{dy}{dx} = -x^2$$

$$\int 5y dy = \int -x^2 dx$$

$$\frac{5}{2}y^2 = -\frac{1}{3}x^3 + C$$

$$\boxed{y^2 = -\frac{2}{15}x^3 + C}$$

$$5) \frac{dr}{ds} = \frac{3}{4}r$$

$$\frac{1}{r} dr = \frac{3}{4} ds$$

$$\ln|r| = \frac{3}{4}s + C$$

$$e^{\ln|r|} = e^{\frac{3}{4}s + C}$$

$$\boxed{r = Ce^{\frac{3}{4}s}}$$

$$7) (2+x)y' = 3y$$

$$(2+x) \frac{dy}{dx} = 3y$$

$$\int \frac{1}{y} dy = \int \frac{3}{2+x} dx$$

$$\ln|y| = 3 \ln|2+x| + C$$

$$e^{\ln|y|} = e^{3 \ln|2+x| + C}$$

$$y = e^{\ln|2+x|^3} \cdot e^C$$

$$\boxed{y = C|2+x|^3}$$

$$9) yy' = 4 \sin x$$

$$y \frac{dy}{dx} = 4 \sin x$$

$$\int y dy = \int 4 \sin x dx$$

$$\frac{1}{2}y^2 = -4 \cos x + C$$

$$\boxed{y^2 = -8 \cos x + C}$$

$$15) yy' - 2e^x = 0$$

$$y \frac{dy}{dx} = 2e^x$$

$$\int y dy = \int 2e^x dx$$

$$\frac{1}{2}y^2 = 2e^x + C$$

$$y^2 = 4e^x + C$$

$$(3)^2 = 4e^0 + C$$

$$9 = 4 + C$$

$$5 = C$$

$$\boxed{y^2 = 4e^x + 5}$$

$$17) y(x+1) + y' = 0$$

$$\frac{dy}{dx} = -y(x+1)$$

$$\int \frac{1}{y} dy = \int -(x+1) dx$$

$$\ln|y| = -\frac{1}{2}x^2 - x + C$$

$$\ln|1| = -\frac{1}{2}(-2)^2 - (-2) + C$$

$$0 = -2 + 2 + C$$

$$0 = C$$

$$\ln|y| = -\frac{1}{2}x^2 - x$$

$$e^{\ln|y|} = e^{-\frac{1}{2}x^2 - x}$$

$$y = e^{-\frac{1}{2}x^2 - x}$$

$$23) dP - kP dt = 0$$

$$dP = kP dt$$

$$\int \frac{1}{P} dP = \int k dt$$

$$\ln|P| = kt + C$$

$$e^{\ln|P|} = e^{kt + C}$$

$$P = Ce^{kt}$$

$$P_0 = Ce^{k(0)}$$

$$P_0 = C e^0$$

$$P_0 = C$$

$$P = P_0 e^{kt}$$

$$25) y' = \frac{x}{4y}$$

$$\frac{dy}{dx} = \frac{x}{4y}$$

$$\int y dy = \int \frac{1}{4} x dx$$

$$\frac{1}{2}y^2 = \frac{1}{8}x^2 + C$$

$$y^2 = \frac{1}{4}x^2 + C$$

$$2^2 = \frac{1}{4}(0)^2 + C$$

$$4 = C$$

$$y^2 = \frac{1}{4}x^2 + 4$$