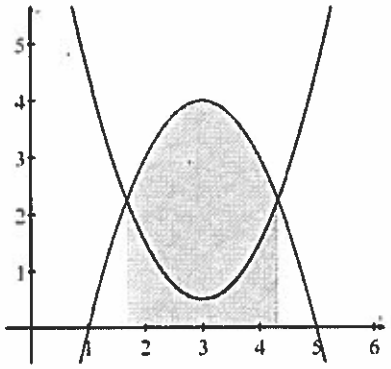
Calculus Section 7.1 Area Between Two Curves

-Find the area of a region between two curves using integration

Homework: page 442 #'s 1-4, 17, 19. 23. 71

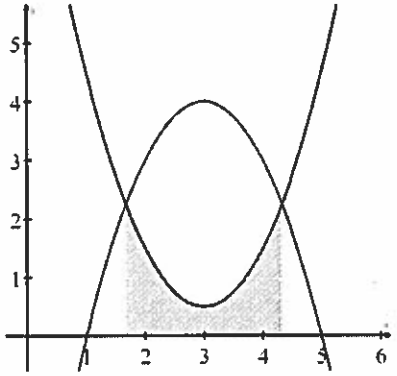
We can extend the idea of definite integrals finding the area of a region *under* a curve to the area of a region *between* two curves. If two functions are both continuous on an interval [a, b], then the region between the curves can be found by subtracting the area of the upper region and the area of the lower region.



Area of upper function

f(x)

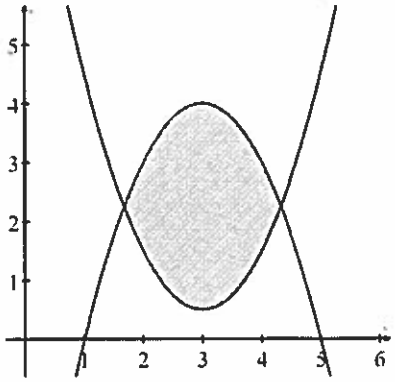
(–)



Area of lower function

g(x)

=



Area between the functions

$$\int_a^b (f(x)-g(x))dx$$

Example) Finding the Area of a Region Between Two Curves

Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.

$$\int_0^1 \left((x^2+2)-(-x)\right)dx$$

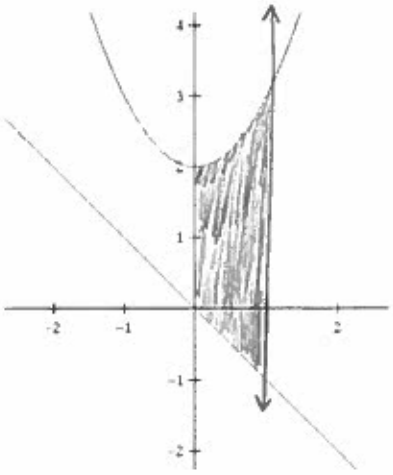
$$\int_0^1 (x^2+x+2)dx$$

$$\left(\frac{1}{3}x^3+\frac{1}{2}x^2+2x\right)\Big|_0^1$$

$$\left(\frac{1}{3}+\frac{1}{2}+2\right)-(0)$$

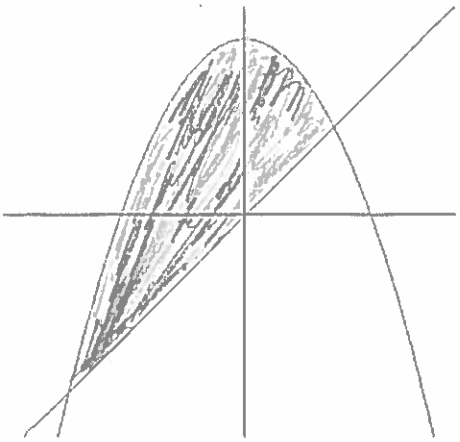
$$\frac{2}{6}+\frac{3}{6}+\frac{12}{6}$$

$$\boxed{\frac{17}{6}}$$



Example) A Region Lying Between Two Intersecting Graphs

Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.



$2 - x^2 = x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$

$$\int_{-2}^{1} \left((2-x^2) - (x)\right) dx$$

$$\int_{-2}^{1} (-x^2 - x + 2)\, dx$$

$$\left(-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right)\Big|_{-2}^{1}$$

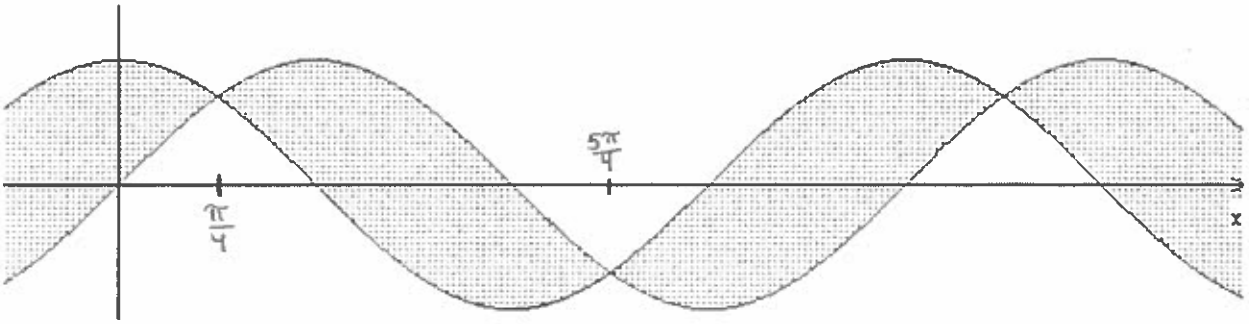
$$\left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$$

$$-\frac{9}{3} - \frac{1}{2} + 8$$

$$-3 - \frac{1}{2} + 8 \rightarrow \boxed{4.5}$$

Example)

The sine and cosine curves intersect infinitely many times, bounding regions of equal areas. Find the area of each one of these regions.



$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots$

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left((\sin x) - (\cos x)\right) dx$$

$$(-\cos x - \sin x)\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$\left(-\cos\left(\frac{5\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right)\right) - \left(-\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\right)$$

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) \rightarrow \frac{4\sqrt{2}}{2} \rightarrow \boxed{2\sqrt{2}}$$