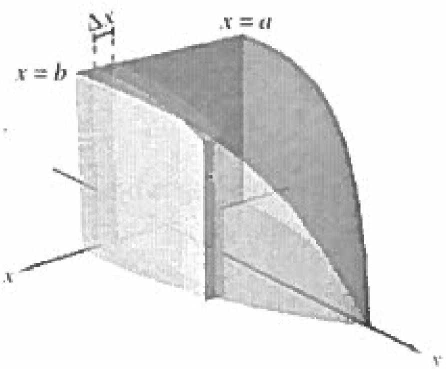
Calculus Section 7.2 Volume by Cross Sections

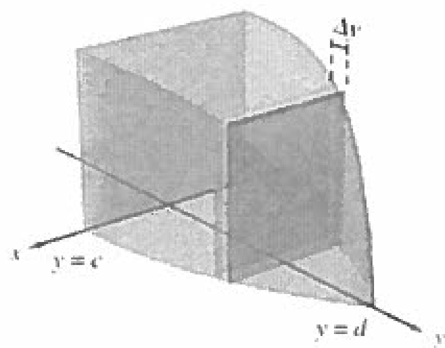
-Find the volume of a solid with known cross sections

Homework: Volume by Cross Sections Worksheet

We have defined the integral of a function as a way to find the ________________________. However, there are other applications of the integral. One application that is related to area is finding the volume of a 3-dimensional surface.

One method to do this involves cross sections. Two-dimensional cross sections can be stacked on top of (or next to) each other to create a 3-d figure. We use the area between two curves to represent the base of the cross sectional shape.





Volumes of Solids with Known Cross Sections

1) For cross sections of area A(x) taken perpendicular to the x-axis: Volume = $\int_a^b A(x)dx$

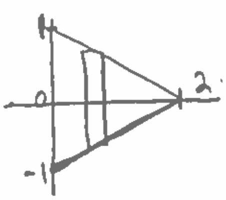
2) For cross sections of area A(y) taken perpendicular to the y-axis: Volume = $\int_c^d A(y)dy$

Where A(x) and A(y) are the known geometric formulas for the area of a shape. For instance, a square cross section has $A(x) = s^2$, a semi-cirlce would be $A(x) = \frac{1}{2}\pi r^2$, and an equilateral triangle is $A(x) = \frac{\sqrt{3}}{4}s^2$.

Example) Use the following region for each cross section.

Find the volume of a triangular shaped solid whose base is the region bounded by the lines f(x) = 1 – x/2, g(x) = -1 + x/2, and x = 0 using...

a) Square cross sections perpendicular to the x-axis

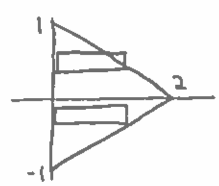


$\int_0^2 (2-x)^2 dx = 2.\overline{6}$

Base: $\left(1-\frac{x}{2}\right)-\left(-1+\frac{x}{2}\right)$

Base: $2-x$

b) Square cross sections perp. to the y-axis



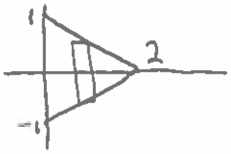
$\int_{-1}^{0} (2y+2)^2 dy + \int_0^1 (2-2y)^2 dy = 2.\overline{6}$

Base: $y = 1-\frac{x}{2} \rightarrow x = 2-2y$

and

Base: $y = -1+\frac{x}{2} \rightarrow x = 2y+2$

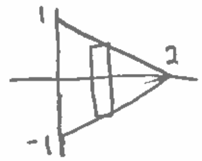
c) Rectangles of h = 4 perp. to the x-axis



Base: $2-x$ height: 4

$$\int_0^2 (2-x)(4)\,dx = 8$$

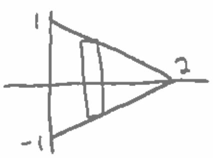
d) Rectangles where h(x) = 4 – x perp. to x-axis



Base: $2-x$ height: $4-x$

$$\int_0^2 (2-x)(4-x)\,dx = 6.\bar{6}$$

e) Semicircles perp. to the x-axis

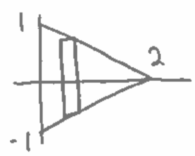


Diameter: $2-x$

Radius: $1-\frac{x}{2}$

$$\frac{\pi}{2}\int_0^2 \left(1-\frac{x}{2}\right)^2 dx = \frac{\pi}{3} \approx 1.047$$

f) Isolsceles right triagles whose leg is bounded by the region and perp. to the x-axis

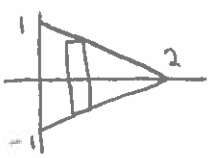


Base: $2-x$

Height: $2-x$

$$\frac{1}{2}\int_0^2 (2-x)(2-x)\,dx = \frac{4}{3}$$

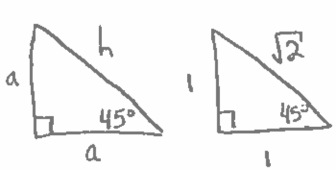
g) Isosceles right triangles where the hypotenuse is bound by the region and perp. to the x-axis.



hypotenuse: $2-x$

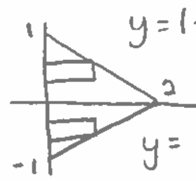
Area $= \frac{1}{2}\left(\frac{h}{\sqrt{2}}\right)^2 \rightarrow \frac{1}{2}\left(\frac{h^2}{2}\right) \rightarrow \frac{1}{4}h^2$

$$\frac{1}{4}\int_0^2 (2-x)^2\,dx = \frac{2}{3}$$



$$\frac{a}{1} = \frac{h}{\sqrt{2}} \rightarrow h = a\sqrt{2} \rightarrow a = \frac{h}{\sqrt{2}}$$

h) Equilateral triangle perp. to the y-axis.



$y = 1-\frac{x}{2} \rightarrow x = 2-2y$

$y = -1+\frac{x}{2} \rightarrow x = 2y+2$

$$\frac{\sqrt{3}}{4}\int_{-1}^{0} (2y+2)^2\,dy + \frac{\sqrt{3}}{4}\int_0^1 (2-2y)^2\,dy = \frac{2\sqrt{3}}{3}$$