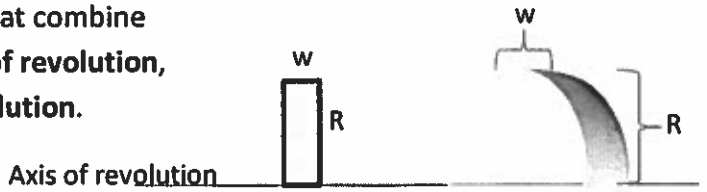
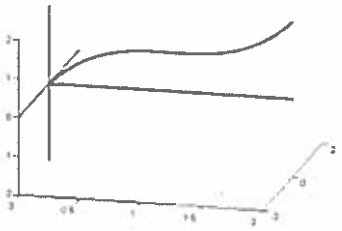
Calculus Section 7.2 Volume by Disk Method

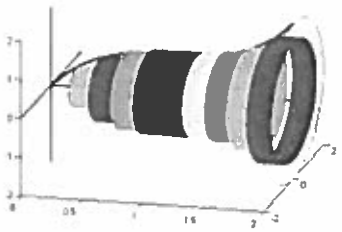
Homework: page 453 #'s 1, 3, 7, 9, 16, 19, 24, 33

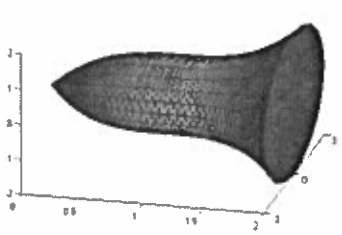
-Find the volume of a solid of revolution using the disk method

Another way to create a 3-dimensional region is by rotating a function around a line. The rotation creates circular cross-sections that combine to create the volume. The resulting solid is called the **solid of revolution**, and the line that it revolved around is called the **axis of revolution**. The area of each circle is $A = \pi r^2$, where r is distance from the function to the axis of revolution.







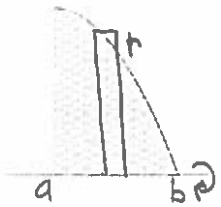


The Disk Method

To find the volume of a solid of revolution with the disk method, use one of the following formulas:

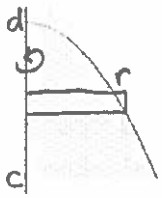
Horizontal Axis of Revolution

$$V = \pi \int_a^b r^2 \, dx$$



Vertical Axis of Revolution

$$V = \pi \int_c^d r^2 \, dy$$



Example) Using the Disk Method x-axis

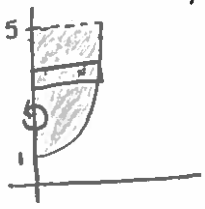
Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis from [0, π] about the x-axis.

$$V = \pi \int_0^{\pi} \left(\sqrt{\sin x}\right)^2 dx$$

$$V = 2\pi$$

Example) Using the Disk Method y-axis

Find the volume of the solid formed by revolving the region bounded by the graph of $y = x^2 + 1$ and the y-axis for $1 \le y \le 5$ about the y-axis.



$$V = \pi \int_1^5 \left(\sqrt{y-1}\right)^2 dy$$

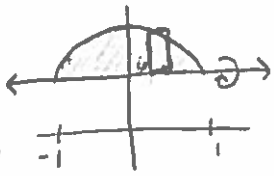
$$x^2 = y - 1$$

$$x = \sqrt{y-1}$$

$$V = 8\pi$$

Example) Revolving About a Line That is Not a Coordinate Axis

Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and g(x) = 1 about the line y = 1.



$r = (2 - x^2) - 1$

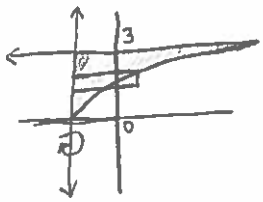
$r = 1 - x^2$

$V = \pi \int_{-1}^{1} (1 - x^2)^2 \, dx$

$V = 1.0\overline{6}\pi$

Example)

Find the volume of the solid formed by revolving the region bounded by $y = \sqrt{x+1}$, y = 3, and x = -1 about the line x = -1.



$r = (y^2 - 1) + 1$

$r = y^2$

$y^2 = x + 1$

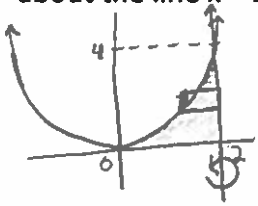
$x = y^2 - 1$

$V = \pi \int_{0}^{3} (y^2)^2 \, dy$

$V = 48.6\pi$

Example)

Find the volume of the solid formed by revolving the region bounded by $y = x^2$, y = 4, the x-axis, and x = 2 about the line x = 2.



$r = 2 - \sqrt{y}$

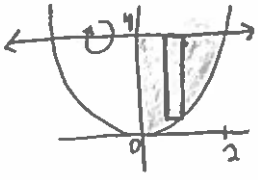
$x = \sqrt{y}$

$V = \pi \int_{0}^{4} (2 - \sqrt{y})^2 \, dy$

$V = 2.66\overline{7}\pi$

Example)

Find the volume of the solid formed by revolving the region bounded by $y = x^2$, y = 4, and the y-axis, about the line y = 4.



$r = 4 - x^2$

$V = \pi \int_{0}^{2} (4 - x^2)^2 \, dx$

$V = 17.0\overline{6}\pi$