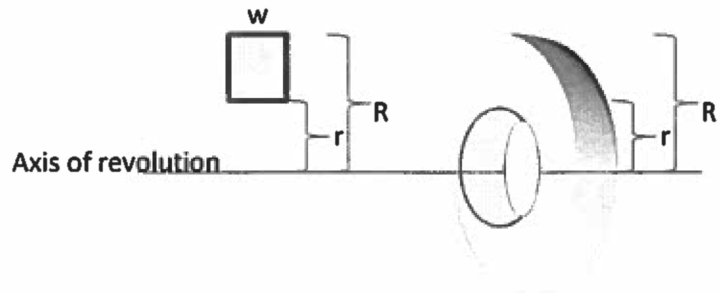
Calculus Section 7.2 Volume by Washer Method

-Find the volume of a solid of revolution using the washer method

Homework: page 453
#'s 5, 6, 11 – 14, 54

The disk method can be extended to cover solids of revolution with holes by replacing the representative disk with a representative **washer**. The washer is formed by revolving a rectangle about an axis.

If r is the inner radius and R is the outer radius, then



$$V = \pi \int_a^b [R(x)^2 - r(x)^2]dx$$

or,

$$V = \pi \int_a^b R(x)^2 dx - \pi \int_a^b r(x)^2 dx$$

Use disks if the region you are rotating lies flush with the axis of rotation. Use washers if there is space between the region and the axis of rotation.

Example) Using the Washer Method

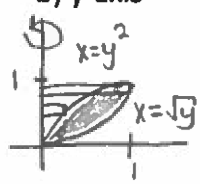
Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the…

a) x-axis.

$$V = \pi \int_0^1 (\sqrt{x})^2 dx - \pi \int_0^1 (x^2)^2 dx$$

$V = .3\pi$

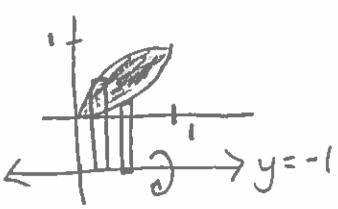
b) y-axis



$$V = \pi \int_0^1 (y^2)^2 dy - \pi \int_0^1 (\sqrt{y})^2 dy$$

$V = .3\pi$

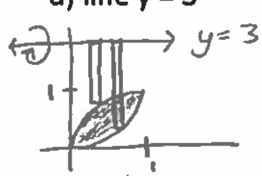
c) line y = -1



$$V = \pi \int_0^1 (\sqrt{x}+1)^2 dx - \pi \int_0^1 (x^2+1)^2 dx$$

$V = 9.667\pi$

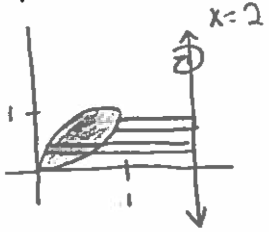
d) line y = 3



$$V = \pi \int_0^1 (3-x^2)^2 dx - \pi \int_0^1 (3-\sqrt{x})^2 dx$$

$V = 1.7\pi$

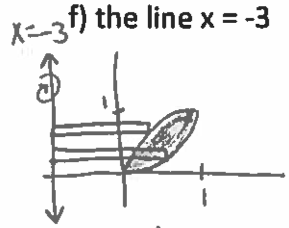
e) the line x = 2



$$V=\pi\int_0^1 (2-y^2)^2\,dy-\pi\int_0^1 (2-\sqrt{y})^2\,dy$$

$$V=1.033\pi$$

f) the line x = -3

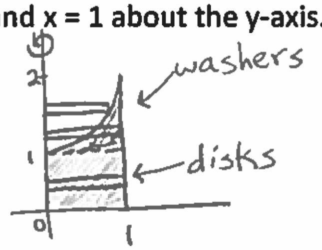


$$V=\pi\int_0^1 (\sqrt{y}+3)^2\,dy-\pi\int_0^1 (y^2+3)^2\,dy$$

$$V=2.3\pi$$

Example) Two-Integral Case

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.

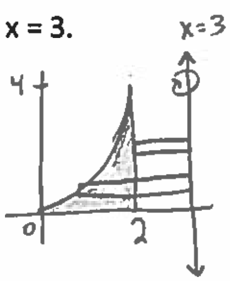


$$V=\underbrace{\pi\int_0^1 1^2\,dy}_{\text{disk}}+\underbrace{\pi\int_1^2 1^2\,dy-\pi\int_1^2 \sqrt{y-1}^{\,2}\,dy}_{\text{washer}}$$

$$V=1.5\pi$$

Example)

Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $y = 0$, and $x = 2$ about the line $x = 3$.



$$V=\pi\int_0^4 (3-\sqrt{y})^2\,dy-\pi\int_0^4 1^2\,dy$$

$$V=8\pi$$