Calculus Section 7.3 Volume by Shells

-Find the volume of a solid of revolution using the shell method
-Compare the uses of the disk method and the shell method

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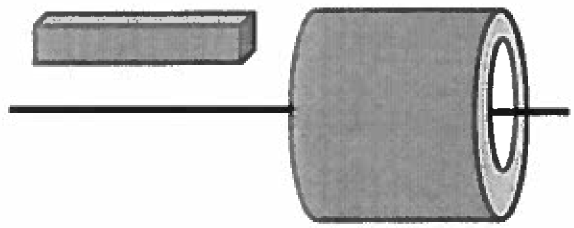
The shell method is an alternative method for finding the volume of a solid of revolution. The shell method uses cylindrical shells to evaluate the volume of a rotation. The shell method is particularly useful when trying to rotate functions that cannot be solved for x around a vertical axis of revolution (i.e. $y = x^3 + 2x^2 - 4x$).

The area of a cylinder is $A = 2\pi rh$, where r is perpendicular to the axis of revolution and h is the length of the cylinder.

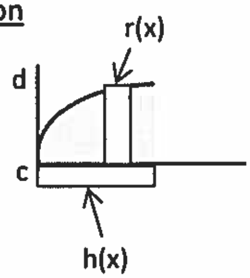


The Shell Method

To find the volume of a solid of revolution with the shell method, use one of the following:

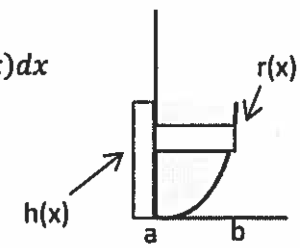
<u>Horizontal Axis of Revolution</u>

$$V = 2\pi \int_c^d r(x)h(x)dx$$



<u>Vertical Axis of Revolution</u>

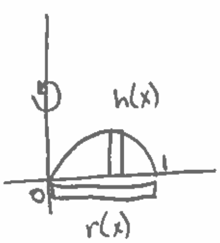
$$V = 2\pi \int_a^b r(x)h(x)dx$$



The limits of integration you use are flipped from the cross-section/disk/washer method. Use the y-values as limits of integration for a horizontal axis of revolution. Use x-values for a vertical axis of revolution.

Example)

Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$, the x-axis, the y-axis, and $x = 1$ about the y-axis.

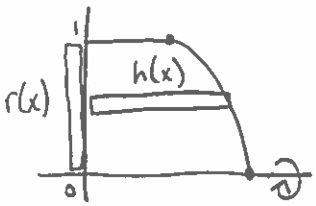


$$V = 2\pi \int_0^1 x(x - x^3)\,dx$$

$$V = .2\bar{6}\pi$$

Example)

Find the volume of the solid of revolution formed by revolving the region bounded by $x = e^{-y^2}$, the x-axis, the y-axis, and y = 1 about the x-axis.

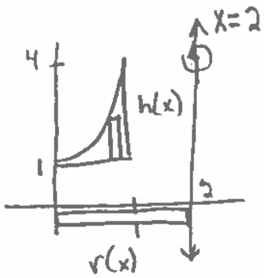


$$V = 2\pi \int_0^1 y\left(e^{-y^2}\right) dy$$

$$V = .632\pi$$

Example)

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, y = 1, and x = 1 about the line x = 2.



$$V = 2\pi \int_0^1 (2-x)(x^3 + x)\, dx$$

$$V = 1.9\bar{3}\,\pi$$

$r(x) = 2 - x$

$h(x) = (x^3 + x + 1) - (1)$