7.4 Arc Length and Surfaces of Revolution

Pg. 473 #'s 3, 11, 13, 21, 38, 43, 44, 47

3) $y = \frac{2}{3}(x^2+1)^{3/2}$

$y' = (x^2+1)^{1/2} \cdot (2x)$

$\text{length} = \int_0^1 \sqrt{1 + ((x^2+1)^{1/2}(2x))^2}\, dx$

length = 1.6̄

11) $y = \ln(\sin x)$

$y' = \frac{1}{\sin x} \cdot \cos x$

$\text{length} = \int_{\pi/4}^{3\pi/4} \sqrt{1 + \left(\frac{\cos x}{\sin x}\right)^2}\, dx$

length = 1.763

13) $y = \frac{1}{2}(e^x + e^{-x})$

$y' = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$\text{length} = \int_0^2 \sqrt{1 + (\frac{1}{2}e^x - \frac{1}{2}e^{-x})^2}\, dx$

length = 3.627

21) $y = \sin x$

$y' = \cos x$

$\text{length} = \int_0^{\pi} \sqrt{1 + (\cos x)^2}\, dx$

length = 3.820

38) $y = 2\sqrt{x}$

$y' = x^{-1/2}$

$S = 2\pi \int_4^9 (2\sqrt{x})\left(\sqrt{1 + (x^{-1/2})^2}\right) dx$

$S = 54.513\pi$

S = 171.258

43) $y = \sqrt[3]{x} + 2$ $\quad x = (y-2)^3$

$x' = 3(y-2)^2$

$S = 2\pi \int_3^4 (y-2)^3 \left(\sqrt{1 + (3(y-2)^2)^2}\right) dy$

$S = 63.497\pi$

S = 199.480

44) $y = 9 - x^2 \rightarrow x = (9-y)^{1/2}$

$x' = \frac{1}{2}(9-y)^{-1/2}(-1)$

$$S = 2\pi \int_0^9 (9-y)^{1/2} \sqrt{1 + \left(\frac{-1}{2}(9-y)^{-1/2}\right)^2}\, dy$$

$S = 37.344\pi$

$\boxed{S = 117.319}$

47) $y = \sin x$

$y' = \cos x$

$$S = 2\pi \int_0^{\pi} \sin x \sqrt{1 + (\cos x)^2}\, dx$$

$S = 4.591\pi$

$\boxed{S = 14.424}$