8.2 Integration by Parts

Pg. 521 #'s 7-11 odd, 23, 25, 27

7) $\int x^3 \ln x\,dx$ $\quad u = \ln x \quad v = \frac{1}{4}x^4 \quad du = \frac{1}{x}dx \quad dv = x^3 dx$

$$\int x^3 \ln x\,dx = (\ln x)\left(\tfrac{1}{4}x^4\right) - \int \tfrac{1}{4}x^4\left(\tfrac{1}{x}dx\right)$$
$$= \tfrac{1}{4}x^4 \ln x - \tfrac{1}{4}\int x^3\,dx$$
$$\boxed{\int x^3 \ln x\,dx = \tfrac{1}{4}x^4 \ln x - \tfrac{1}{16}x^4 + C}$$

9) $\int x \sin 3x\,dx$ $\quad u = x \quad v = -\frac{1}{3}\cos 3x \quad du = dx \quad dv = \sin 3x\,dx$

$$\int x\sin 3x\,dx = x\left(-\tfrac{1}{3}\cos 3x\right) - \int -\tfrac{1}{3}\cos 3x\,dx$$
$$= -\tfrac{1}{3}x\cos 3x + \tfrac{1}{3}\int \cos 3x\,dx$$
$$\boxed{\int x\sin 3x\,dx = -\tfrac{1}{3}x\cos 3x + \tfrac{1}{9}\sin 3x + C}$$

11) $\int x e^{-4x}\,dx$ $\quad u = x \quad v = -\frac{1}{4}e^{-4x} \quad du = dx \quad dv = e^{-4x}dx$

$$\int x e^{-4x}dx = x\left(-\tfrac{1}{4}e^{-4x}\right) - \int -\tfrac{1}{4}e^{-4x}\,dx$$
$$= -\tfrac{1}{4}xe^{-4x} + \tfrac{1}{4}\int e^{-4x}\,dx$$
$$\boxed{\int x e^{-4x}dx = -\tfrac{1}{4}xe^{-4x} - \tfrac{1}{16}e^{-4x} + C}$$

23) $\int x\cos x\,dx$ $\quad u = x \quad v = \sin x \quad du = dx \quad dv = \cos x\,dx$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$
$$\boxed{\int x\cos x\,dx = x\sin x + \cos x + C}$$

25) $\int x^3 \sin x\,dx$

$u = x^3 \quad v = -\cos x$

$du = 3x^2 dx \quad dv = \sin x\,dx$

$\int x^3 \sin x\,dx = -x^3\cos x - \int -\cos x\,(3x^2 dx)$

$= -x^3\cos x + \int 3x^2 \cos x\,dx$

$u = 3x^2 \quad v = \sin x$

$du = 6x\,dx \quad dv = \cos x\,dx$

$\int 3x^2\cos x\,dx = 3x^2 \sin x - \int \sin x\,(6x\,dx)$

$= 3x^2\sin x + \int (-6x)\sin x\,dx$

$u = -6x \quad v = -\cos x$

$du = -6dx \quad dv = \sin x\,dx$

$\int -6x \sin x\,dx = 6x\cos x - \int -\cos x\,(-6dx)$

$= 6x\cos x + 6\int \cos x\,dx$

$= 6x\cos x + 6\sin x$

$$\boxed{\int x^3 \sin x\,dx = -x^3\cos x + 3x^2\sin x + 6x\cos x + 6\sin x}$$

27) $\int \arctan x\,dx$

$u = \arctan x \quad v = x$

$du = \frac{1}{1+x^2}dx \quad dv = dx$

$\int \arctan x\,dx = x\arctan x - \int \frac{x}{1+x^2}dx$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int \frac{1}{u}du$

$-\frac{1}{2}\ln|1+x^2|$

$$\boxed{\int \arctan x\,dx = x\arctan x - \frac{1}{2}\ln|1+x^2| + C}$$