8.2 Integration by Parts II

Pg. 521 #'s 29, 30, 39, 41, 49, 53

29) $\int e^{-3x}\sin 5x\,dx$ $\qquad u=\sin 5x \quad v=-\frac{1}{3}e^{-3x}$

$\qquad du=5\cos 5x\,dx \quad dv=e^{-3x}\,dx$

$\int e^{-3x}\sin 5x\,dx=-\frac{1}{3}e^{-3x}\sin 5x+\int +\frac{5}{3}e^{-3x}\cos 5x\,dx$ $\qquad u=\cos 5x \quad v=\frac{-5}{9}e^{-3x}$

$\qquad du=-5\sin 5x\,dx \quad dv=\frac{5}{3}e^{-3x}\,dx$

$$\int \frac{5}{3}e^{-3x}\cos 5x\,dx=-\frac{5}{9}e^{-3x}\cos 5x-\int \frac{25}{9}e^{-3x}\sin 5x\,dx$$

$$\int e^{-3x}\sin 5x\,dx=-\frac{1}{3}e^{-3x}\sin 5x-\frac{5}{9}e^{-3x}\cos 5x-\frac{25}{9}\int e^{-3x}\sin 5x\,dx$$

$$\frac{34}{9}\int e^{-3x}\sin 5x=-\frac{1}{3}e^{-3x}\sin 5x-\frac{5}{9}e^{-3x}\cos 5x$$

$$\boxed{\int e^{-3x}\sin 5x=-\frac{3}{34}e^{-3x}\sin 5x-\frac{5}{34}e^{-3x}\cos 5x+C}$$

30) $\int e^{4x}\cos 2x\,dx$ $\qquad u=\cos 2x \quad v=\frac{1}{4}e^{4x}$

$\qquad du=-2\sin 2x\,dx \quad dv=e^{4x}\,dx$

$\int e^{4x}\cos 2x\,dx=\frac{1}{4}e^{4x}\cos 2x+\int +\frac{1}{2}e^{4x}\sin 2x\,dx$ $\qquad u=\sin 2x \quad v=\frac{1}{8}e^{4x}$

$\qquad du=2\cos 2x\,dx \quad dv=\frac{1}{2}e^{4x}\,dx$

$$\int \frac{1}{2}e^{4x}\sin 2x\,dx=\frac{1}{8}e^{4x}\sin 2x-\int \frac{1}{4}e^{4x}\cos 2x\,dx$$

$$\int e^{4x}\cos 2x\,dx=\frac{1}{4}e^{2x}\cos 2x+\frac{1}{8}e^{4x}\sin 2x-\frac{1}{4}\int e^{4x}\cos 2x\,dx$$

$$\frac{5}{4}\int e^{4x}\cos 2x\,dx=\frac{1}{4}e^{2x}\cos 2x+\frac{1}{8}e^{4x}\sin 2x$$

$$\boxed{\int e^{4x}\cos 2x\,dx=\frac{1}{5}e^{2x}\cos 2x+\frac{1}{10}e^{4x}\sin 2x+C}$$

39) $\int_0^3 xe^{x/2}\,dx$

signs	u	dv
+	x	$e^{\frac{1}{2}x}$
−	1	$2e^{\frac{1}{2}x}$
+	0	$4e^{\frac{1}{2}x}$

$$\left[2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right]_0^3$$

$$\left(6e^{3/2} - 4e^{3/2}\right) - \left(0 - 4e^0\right)$$

$$\boxed{2e^{3/2} + 4}$$

41) $\int_0^{\pi/4} x\cos 2x\,dx$

signs	u	dv
+	x	$\cos 2x$
−	1	$\frac{1}{2}\sin 2x$
+	0	$\frac{1}{4}\cos 2x$

$$\left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\pi/4}$$

$$\left(\frac{\pi}{8}\sin\left(\frac{\pi}{2}\right) + \frac{1}{4}\cos\left(\frac{\pi}{2}\right)\right) - \left(0 + \frac{1}{4}\cos(0)\right)$$

$$\frac{\pi}{8} + 0 - 0 - \frac{1}{4}$$

$$\boxed{\frac{\pi}{8} - \frac{1}{4} \approx .143}$$

49) $\int x^2 e^{2x}\,dx$

signs	u	dv
+	x^2	e^{2x}
−	$2x$	$\frac{1}{2}e^{2x}$
+	2	$\frac{1}{4}e^{2x}$
−	0	$\frac{1}{8}e^{2x}$

$$\boxed{\int x^2 e^{2x}\,dx = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C}$$

53) $\int x\sec^2 x\,dx$

signs	u	dv
+	x	$\sec^2 x$
−	1	$\tan x$
+	0	$-\ln\lvert\cos x\rvert$

$$\boxed{\int x\sec^2 x\,dx = x\tan x + \ln\lvert\cos x\rvert + C}$$