# Calculus Section 8.2 Integration by Parts II

-Use tabular method to perform integration by parts

Homework: page 521 #'s 29 or 30, 39, 41, 49, 53

**In problems that require repeated application of the integration by parts method, a tabular method can help organize and speed up the pace of the problem.**

**Example) Use integration by parts**

Find $\int x^2 \sin(4x)\,dx$ $u = x^2$ $v = -\frac{1}{4}\cos 4x$

$du = 2x$ $dv = \sin 4x\,dx$

$\int x^2 \sin 4x\,dx = -\frac{1}{4}x^2\cos 4x + \int +\frac{1}{2}x\cos 4x\,dx$ $u = \frac{1}{2}x$ $v = \frac{1}{4}\sin 4x$

$du = \frac{1}{2}dx$ $dv = \cos 4x\,dx$

$\int \frac{1}{2}x\cos 4x\,dx = \frac{1}{8}x\sin 4x - \int \frac{1}{8}\sin 4x\,dx$

$\int \frac{1}{2}x\cos 4x\,dx = \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x$

$$\int x^2 \sin 4x\,dx = -\frac{1}{4}x^2\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x + C$$

**Example) Use tabular method**

Find $\int x^2 \sin(4x)\,dx$

| signs | u | dv |
|---|---|---|
| + → | $x^2$ | $\sin 4x$ |
| − → | $2x$ | $-\frac{1}{4}\cos 4x$ |
| + → | $2$ | $-\frac{1}{16}\sin 4x$ |
| − | $0$ | $\frac{1}{64}\cos 4x$ |

$$\int x^2 \sin(4x)\,dx = -\frac{1}{4}x^2\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x + C$$

**Example) Definite Integral**

Find $\int_0^1 (x^2 - 1)e^x dx$

| signs | $u$ | $dv$ |
|---|---|---|
| + | $x^2-1$ | $e^x$ |
| − | $2x$ | $e^x$ |
| + | $2$ | $e^x$ |
| − | $0$ | |

$$\left[(x^2-1)e^x - 2xe^x + 2e^x\right]_0^1$$

$$(0 - 2e + 2e) - (-1 - 0 + 2)$$

$$\boxed{-1}$$

**Example) Non-terminating factors**

Find $\int e^x \sin(x)\, dx$

$u = \sin x \qquad v = e^x$

$du = \cos x\, dx \qquad dv = e^x dx$

$$\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx$$

$u = \cos x \qquad v = e^x$

$du = -\sin x\, dx \qquad dv = e^x dx$

$$\int e^x \cos x\, dx = e^x \cos x + \int + e^x \sin x\, dx$$

$$\int e^x \sin x\, dx = e^x \sin x - \left(e^x \cos x + \int e^x \sin x\, dx\right)$$

$$\int e^x \sin x\, dx = e^x \sin x - e^x \cos x - \int e^x \sin x\, dx$$

$$+ \int e^x \sin x\, dx$$

$$2\int e^x \sin x\, dx = e^x \sin x - e^x \cos x$$

$$\boxed{\int e^x \sin x\, dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + C}$$