Calculus Section 8.2 Integration by Parts

-Find an antiderivative using integration by parts

Homework: page 521 #'s
7 – 11 odd, 23 – 27 odd

Integration by parts is a technique that can be applied to a wide variety of functions but is particularly helpful when you have an integral involving the product of two functions. Integration by parts is the first secondary resource you should consult if you cannot fit an integral to one of the basic rules.

Integration by parts is based on the formula for the derivative of a product (product rule):

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \text{where } dv = \frac{dv}{dx} \text{ and } du = \frac{du}{dx}$$

$$\int \frac{d}{dx}uv = \int u\,dv + \int v\,du$$

$$uv = \int u\,dv + \int v\,du$$

$$uv - \int v\,du = \int u\,dv \longrightarrow \int u\,dv = uv - \int v\,du$$

Integration by Parts

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du$$

This formula expresses the original integral in terms of another integral. Depending on the choices of u and dv, it may be easier to evaluate the second integral than the original one.

Guidelines for integration by parts

Let dv be a function that can be integrated. Let u be a function whose derivative is typically simpler than u.

Example)

Find $\int xe^x\,dx$ $\quad u = x \quad v = e^x$

$du = dx \quad dv = e^x dx$

$$\int xe^x dx = xe^x - \int e^x dx$$

$$\boxed{\int xe^x dx = xe^x - e^x + C}$$

we don't know the integral of $\ln x$, so it must be u

Find $\int x^2 \ln x \, dx$

$u = \ln x \qquad v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx \qquad dv = x^2 dx$

$$\int x^2 \ln x \, dx = \ln x\left(\tfrac{1}{3}x^3\right) - \int \tfrac{1}{3}x^3\left(\tfrac{1}{x}\right)dx$$

$$= \tfrac{1}{3}x^3 \ln x - \tfrac{1}{3}\int x^2 dx$$

$$\boxed{\int x^2 \ln x \, dx = \tfrac{1}{3}x^3 \ln x - \tfrac{1}{9}x^3 + C}$$

Find $\int \ln x \, dx$

$u = \ln x \qquad v = x$

$du = \frac{1}{x} \qquad dv = 1\,dx$

$$\int \ln x \, dx = (\ln x)(x) - \int x\left(\tfrac{1}{x}\right)dx$$

$$= x \ln x - \int dx$$

$$\boxed{\int \ln x \, dx = x \ln x - x + C}$$

Repeated Use of Integration by Parts

Find $\int x^2 \sin x \, dx$

$u = x^2 \qquad v = -\cos x$

$du = 2x\,dx \qquad dv = \sin x\,dx$

$$\int x^2 \sin x \, dx = x^2(-\cos x) - \int (-\cos x)2x\,dx$$

$$= -x^2 \cos x + \int 2x \cos x \, dx$$

$u = 2x \qquad v = \sin x$

$du = 2dx \qquad dv = \cos x\,dx$

$$\int 2x \cos x \, dx = 2x \sin x - \int 2 \sin x \, dx$$

$$= 2x \sin x + 2\cos x + C$$

$$\boxed{\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C}$$