8.3 Trig Functions with Powers

Pg. 530 #'s 1, 3, 5, 9, 21, 25, 27

1) $\int \cos^5 x \sin x \, dx$

$\int \cos x (\cos^2 x)^2 \sin x \, dx$

$\int \cos x (1-\sin^2 x)^2 \sin x \, dx$

$\int \cos x (1 - 2\sin^2 x + \sin^4 x) \sin x \, dx$

$\int \cos x (\sin x - 2\sin^3 x + \sin^5 x) \, dx$

$\int \sin x \cos x \, dx - 2\int \sin^3 x \cos x \, dx + \int \sin^5 x \cos x \, dx$

$u = \sin x \quad du = \cos x \, dx$

$\int u \, du - 2\int u^3 \, du + \int u^5 \, du$

$\frac{1}{2}u^2 - \frac{2}{4}u^4 + \frac{1}{6}u^6$

$\boxed{\frac{1}{2}\sin^2 x - \frac{1}{2}\sin^4 x + \frac{1}{6}\sin^6 x + C}$

3) $\int \sin^7 2x \cos 2x \, dx$

$u = \sin 2x$

$du = 2\cos 2x \, dx$

$\frac{1}{2}\int u^7 \, du$

$\frac{1}{16}u^8$

$\boxed{\frac{1}{16}\sin^8 2x + C}$

5) $\int \sin^3 x \cos^2 x \, dx$

$\int \sin x (\sin^2 x) \cos^2 x \, dx$

$\int \sin x (1 - \cos^2 x) \cos^2 x \, dx$

$\int \sin x (\cos^2 x - \cos^4 x) \, dx$

$\int \cos^2 x \sin x \, dx - \int \cos^4 x \sin x \, dx$

$u = \cos x \quad du = -\sin x \, dx$

$-\int u^2 \, du + \int u^4 \, du$

$-\frac{1}{3}u^3 + \frac{1}{5}u^5$

$\boxed{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C}$

9) $\int \cos^2 3x \, dx$

$\int \frac{1+\cos 6x}{2} \, dx$

$\int \frac{1}{2} \, dx + \int \frac{1}{2}\cos 6x \, dx$

$\boxed{\frac{1}{2}x + \frac{1}{12}\sin 6x + C}$

21) $\int \sec^3 \pi x \, dx$ $\quad u = \sec \pi x$ $\quad v = \frac{1}{\pi} \tan \pi x$

$du = \pi \sec \pi x \tan \pi x$ $\quad dv = \sec^2 \pi x$

$$\int \sec^3 \pi x \, dx = \frac{1}{\pi} \sec \pi x \tan \pi x - \int \sec \pi x \tan^2 \pi x \, dx$$

$$- \int \sec \pi x (\sec^2 \pi x - 1) dx$$

$$- \int \sec^3 \pi x \, dx + \int \sec \pi x \, dx$$

$$\int \sec^3 \pi x \, dx = \frac{1}{\pi} \sec \pi x \tan \pi x - \int \sec^3 \pi x \, dx + \frac{1}{\pi} \ln |\sec \pi x + \tan \pi x|$$

$$2 \int \sec^3 \pi x \, dx = \frac{1}{\pi} \sec \pi x \tan \pi x + \frac{1}{\pi} \ln |\sec \pi x + \tan \pi x|$$

$$\boxed{\int \sec^3 \pi x \, dx = \frac{1}{2\pi} \sec \pi x \tan \pi x + \frac{1}{2\pi} \ln |\sec \pi x + \tan \pi x| + C}$$

25) $\int \tan^3 2t \sec^3 2t \, dt$

$$\int (\sec 2t \tan 2t)(\tan^2 2t)(\sec^2 2t) dt$$

$$\int (\sec 2t \tan 2t)(\sec^2 2t - 1)(\sec^2 2t) dt$$

$$\int (\sec 2t \tan 2t)(\sec^4 2t - \sec^2 2t) dt$$

$$\int \sec^4 2t (\sec 2t \tan 2t) dt - \int \sec^2 2t (\sec 2t \tan 2t) dt$$

$u = \sec 2t \quad du = 2 \sec 2t \tan 2t$

$$\frac{1}{2} \int u^4 \, du - \frac{1}{2} \int u^2 \, du$$

$$\frac{1}{10} u^5 - \frac{1}{6} u^3$$

$$\boxed{\frac{1}{10} \sec^5 2t - \frac{1}{6} \sec^3 2t + C}$$

27) $\int \sec^6 4x \tan 4x \, dx$

$$\int \sec^2 4x (\sec^2 4x)^2 \tan 4x \, dx$$

$$\int \sec^2 4x (1 + \tan^2 4x)^2 \tan 4x \, dx$$

$$\int \sec^2 4x (1 + 2\tan^2 4x + \tan^4 4x) \tan 4x \, dx$$

$$\int \tan 4x \sec^2 4x \, dx + 2 \int \tan^3 4x \sec^2 4x \, dx + \int \tan^5 4x \sec^2 4x \, dx$$

$u = \tan 4x \quad du = 4 \sec^2 4x$

$$\frac{1}{4} \int u \, du + \frac{2}{4} \int u^3 \, du + \frac{1}{4} \int u^5 \, du$$

$$\frac{1}{8} u^2 + \frac{1}{8} u^4 + \frac{1}{24} u^6$$

$$\boxed{\frac{1}{8} \tan^2 4x + \frac{1}{8} \tan^4 4x + \frac{1}{24} \tan^6 4x + C}$$