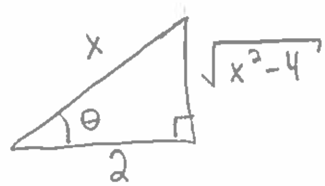
8.4 Trig Substitution

Pg. 539 #'s 24, 25, 30

24) $\int \frac{1}{\sqrt{x^2-4}}\,dx$



$u = 2\sec\theta$

$du = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-4} = 2\tan\theta$

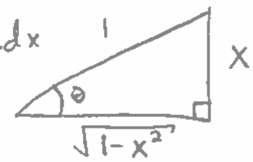
$\int \frac{1}{2\tan\theta}\, 2\sec\theta\tan\theta\,d\theta$

$\int \sec\theta\,d\theta$

$\ln|\sec\theta + \tan\theta|$

$\boxed{\ln\left|\frac{1}{2}x + \frac{1}{2}\sqrt{x^2-4}\right| + C}$

25) $\int \frac{\sqrt{1-x^2}}{x^4}\,dx$



$u = \sin\theta$

$du = \cos\theta\,d\theta$

$\sqrt{1-x^2} = \cos\theta$

$\int \frac{\cos\theta}{\sin^4\theta}\cos\theta\,d\theta$

$\int \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\sin^2\theta}\,d\theta$

$\int \cot^2\theta\csc^2\theta\,d\theta$

$u = \cot\theta$

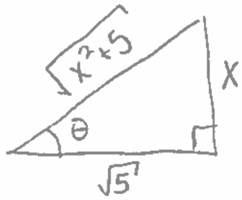
$du = -\csc^2\theta\,d\theta$

$-\int u^2\,du$

$-\frac{1}{3}u^3$

$-\frac{1}{3}\cot^3\theta \rightarrow \boxed{-\frac{1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^3 + C}$

30) $\int \frac{1}{(x^2+5)^{3/2}}dx$



$u=\sqrt{5}\tan\theta$

$du=\sqrt{5}\sec^2\theta d\theta$

$\sqrt{x^2+5} = \sqrt{5}\sec\theta$

$\int \frac{1}{(\sqrt{5}\sec\theta)^3}\sqrt{5}\sec^2\theta d\theta$

$\int \frac{\sqrt{5}\sec^2\theta}{(\sqrt{5})^3\sec^3\theta}d\theta$

$\int \frac{1}{(\sqrt{5})^2\sec\theta}d\theta$

$\frac{1}{5}\int \cos\theta \, d\theta$

$\frac{1}{5}\sin\theta$

$\frac{1}{5}\left(\frac{X}{\sqrt{X^2+5}}\right)$

$\boxed{\frac{X}{5\sqrt{x^2+5}} + C}$