

8.5 Partial Fractions (Linear Factors)

Pg. 549 #'s 5, 7, 9, 15, 23

$$5) \int \frac{1}{x^2-9} dx = \frac{A}{x+3} + \frac{B}{x-3}$$

$$1 = A(x-3) + B(x+3)$$

Let $x=3$

$$1 = A(0) + B(6)$$

$$1 = 6B$$

$$\frac{1}{6} = B$$

Let $x=-3$

$$1 = A(-6) + B(0)$$

$$1 = -6A$$

$$-\frac{1}{6} = A$$

$$-\frac{1}{6} \int \frac{1}{x+3} dx + \frac{1}{6} \int \frac{1}{x-3} dx$$

$$-\frac{1}{6} \ln|x+3| + \frac{1}{6} \ln|x-3| + C$$

$$7) \int \frac{5}{x^2+3x-4} dx = \frac{A}{x+4} + \frac{B}{x-1}$$

$$5 = A(x-1) + B(x+4)$$

Let $x=1$

$$5 = A(0) + B(5)$$

$$5 = 5B$$

$$1 = B$$

Let $x=-4$

$$5 = A(-5) + B(0)$$

$$5 = -5A$$

$$-1 = A$$

$$-\int \frac{1}{x+4} dx + \int \frac{1}{x-1} dx$$

$$-\ln|x+4| + \ln|x-1| + C$$

$$9) \int \frac{x^2+12x+12}{x^3-4x} dx = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$$

$$x(x^2-4)$$

$$x^2+12x+12 = A(x+2)(x-2) + B(x)(x-2) + C(x)(x+2)$$

Let $x=0$

$$12 = A(2)(-2) + B(0)(-2) + C(0)(2)$$

$$12 = -4A$$

$$-3 = A$$

Let $x=2$

$$40 = A(4)(0) + B(2)(0) + C(2)(4)$$

$$40 = 8C$$

$$5 = C$$

Let $x=-2$

$$-8 = A(0)(-4) + B(-2)(-4) + C(-2)(0)$$

$$-8 = 8B$$

$$-1 = B$$

$$-3 \int \frac{1}{x} dx - \int \frac{1}{x+2} dx + 5 \int \frac{1}{x-2} dx$$

$$-3 \ln|x| - \ln|x+2| + 5 \ln|x-2| + C$$

$$15) \int \frac{x^2+3x-4}{x^3-4x^2+4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$x(x^2-4x+4)$
 $x(x-2)(x-2)$

$$x^2+3x-4 = A(x-2)^2 + B(x)(x-2) + C(x)$$

Let $x=0$

$$-4 = A(-2)^2 + B(0)(-2) + C(0)$$

$$-4 = 4A$$

$$-1 = A$$

Let $x=2$

$$6 = A(0)^2 + B(2)(0) + C(2)$$

$$6 = 2C$$

$$3 = C$$

Let $x=1$

$$0 = A(-1)^2 + B(1)(-1) + C(1)$$

$$0 = A - B + C$$

$$0 = -1 - B + 3$$

$$B = 2$$

$$-\int \frac{1}{x} dx + 2 \int \frac{1}{x-2} dx + 3 \int \frac{1}{(x-2)^2} dx$$

$u = x-2$
 $du = dx$

$$-\ln|x| + 2 \ln|x-2| + 3 \int u^{-2} du$$

$$-\ln|x| + 2 \ln|x-2| - 3 \frac{1}{x-2} + C$$

$$23) \int_0^2 \frac{3}{4x^2+5x+1} dx = \frac{A}{4x+1} + \frac{B}{x+1}$$

$(4x+1)(x+1)$

$$3 = A(x+1) + B(4x+1)$$

Let $x = -1$

$$3 = A(0) + B(-3)$$

$$3 = -3B$$

$$-1 = B$$

Let $x = -1/4$

$$3 = A(3/4) + B(0)$$

$$3 = \frac{3}{4}A$$

$$4 = A$$

$$4 \int_0^2 \frac{1}{4x+1} dx - \int_0^2 \frac{1}{x+1} dx$$

$u = 4x+1$

$du = 4dx$

$\frac{1}{4} du = dx$

$$\int \frac{1}{u} du - \ln|x+1|$$

$$\left[\ln|4x+1| - \ln|x+1| \right]_0^2$$

$$(\ln|9| - \ln|3|) - (\ln|1| - \ln|1|)$$

$$\ln|9| - \ln|3|$$

$$\boxed{\ln|3|}$$