Calculus Section 8.5 Partial Fractions (Linear Factors)

-Understand the concept of partial fraction decomposition
-Use partial faction decomposition with linear functions

Homework: page 549 #'s 5, 7, 9, 15, 23

Partial fractions is used to integrate rational functions with polynomial denominators that cannot be integrated with simple u-substitution (i.e. $\int \frac{1}{x^2-5x+6}dx$). These types of functions can be integrated by completing the square and using trigonometric substitution. Partial fractions is a simpler method that re-writes the rational function as the sum/difference of multiple fractions $\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{x-3}dx - \int \frac{1}{x-2}dx$.

Example)

$$\int \frac{1}{x^2-5x+6}dx = \frac{A}{x-3} + \frac{B}{x-2}$$

$(x-3)(x-2)$

$1 = A(x-2) + B(x-3)$

Let $x = 2$

$1 = A(0) + B(-1)$

$1 = -B$

$-1 = B$

Let $x = 3$

$1 = A(1) + B(0)$

$1 = A$

$\int \frac{1}{x-3}dx - \int \frac{1}{x-2}dx$

$\ln|x-3| - \ln|x-2| + C$

$$\boxed{\ln\left|\frac{x-3}{x-2}\right| + C}$$

$$\int \frac{5x-1}{x^3-9x}dx = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-3}$$

$x(x^2-9)$

$x(x+3)(x-3)$

$5x-1 = A(x+3)(x-3) + B(x)(x-3) + C(x)(x+3)$

Let $x = 0$

$-1 = A(3)(-3) + B(0)(-3) + C(0)(3)$

$-1 = -9A$

$\frac{1}{9} = A$

Let $x = 3$

$14 = A(9)(0) + B(3)(0) + C(3)(6)$

$14 = 18C$

$\frac{7}{9} = C$

Let $x = -3$

$-16 = A(0)(-6) + B(-3)(-6) + C(-3)(0)$

$-16 = 18B$

$-\frac{8}{9} = B$

$$\frac{1}{9}\int \frac{1}{x}dx - \frac{8}{9}\int \frac{1}{x+3}dx + \frac{7}{9}\int \frac{1}{x-3}dx \rightarrow \boxed{\frac{1}{9}\ln|x| - \frac{8}{9}\ln|x+3| + \frac{7}{9}\ln|x-3| + C}$$

Example) Repeated Linear Factors (include a fraction for each power of a factor)

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$x(x^2+2x+1)$

$x(x+1)(x+1)$

$x(x+1)^2$

$5x^2+20x+6 = A(x+1)^2 + B(x)(x+1) + C(x)$

Let $x=0$

$6 = A(1)^2 + B(0)(1) + C(0)$

$6 = A$

Let $x=-1$

$-9 = A(0)^2 + B(-1)(0) + C(-1)$

$-9 = -C$

$9 = C$

Let $x=1$

$31 = A(2)^2 + B(1)(2) + C(1)$

$31 = 4A + 2B + C$

$31 = 4(6) + 2B + 9$

$31 = 24 + 2B + 9$

$31 = 33 + 2B$

$-2 = 2B$

$-1 = B$

$6\int \frac{1}{x}dx - \int \frac{1}{x+1}dx + 9\int \frac{1}{(x+1)^2}dx$

$u = x+1$

$du = dx$

$6\ln|x| - \ln|x+1| + 9\int u^{-2}du$

$6\ln|x| - \ln|x+1| - 9u^{-1}$

$$\boxed{6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C}$$