

8.5 Partial Fractions (Quadratic Factors)

Pg. 549 #'s 12, 17, 19

$$12) \int \frac{x+2}{x^2+5x} dx = \frac{A}{x} + \frac{B}{x+5}$$

$$x+2 = A(x+5) + B(x)$$

Let $x=0$
 $2 = 5A + B(0)$

Let $x=-5$
 $-3 = A(0) - 5B$
 $\frac{3}{5} = B$

$$\int \frac{x+2}{x^2+5x} dx = \frac{2}{5} \int \frac{1}{x} dx + \frac{3}{5} \int \frac{1}{x+5} dx \quad \frac{2}{5} = A$$

$$\boxed{\frac{2}{5} \ln|x| + \frac{3}{5} \ln|x+5| + C}$$

$$17) \int \frac{x^2-1}{x^3+x} dx = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$x(x^2+1)$

$$x^2-1 = A(x^2+1) + (Bx+C)(x)$$

Let $x=0$

$$-1 = A(1) + (B(0)+C)(0)$$

$$-1 = A$$

Let $x=1$

$$0 = A(2) + (B+C)(1)$$

$$0 = 2A + B + C$$

$$0 = -2 + B + C$$

$$2 = B + C$$

Let $x=-1$

$$0 = A(2) + (-B+C)(-1)$$

$$0 = 2A + B - C$$

$$0 = -2 + B - C$$

$$2 = B - C$$

$$\begin{cases} B-C=2 \\ B+C=2 \end{cases} \quad B=2, C=0$$

$$-\int \frac{1}{x} dx + \int \frac{2x}{x^2+1} dx$$

$$u = x^2+1$$

$$du = 2x dx$$

$$-\int \frac{1}{x} dx + \int \frac{1}{u} du$$

$$-\ln|x| + \ln|x^2+1| + C$$

$$\boxed{\ln \left| \frac{x^2+1}{x} \right| + C}$$

$$19) \int \frac{x^2}{x^4 - 2x^2 - 8} dx = \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+2}$$

$$(x^2-4)(x^2+2)$$

$$(x+2)(x-2)(x^2+2)$$

$$x^2 = A(x-2)(x^2+2) + B(x+2)(x^2+2) + (Cx+D)(x-2)(x+2)$$

$$\text{Let } x=2$$

$$4 = A(0)(6) + B(4)(6) + (2C+2)(0)(4)$$

$$4 = 24B$$

$$\frac{1}{6} = B$$

$$\text{Let } x=-2$$

$$4 = A(-4)(6) + B(0)(6) + (-2C+D)(-4)(0)$$

$$4 = -24A$$

$$-\frac{1}{6} = A$$

$$\text{Let } x=0$$

$$0 = A(-2)(2) + B(2)(2) + (C(0)+D)(-2)(2)$$

$$0 = -4A + 4B - 4D$$

$$0 = \frac{2}{3} + \frac{2}{3} - 4D$$

$$-\frac{4}{3} = -4D$$

$$\frac{1}{3} = D$$

$$\text{Let } x=1$$

$$1 = A(-1)(3) + B(3)(3) + (C+D)(-1)(3)$$

$$1 = -3A + 9B - 3C - 3D$$

$$1 = \frac{1}{2} + \frac{3}{2} - 3C - 1$$

$$2 = 2 - 3C$$

$$0 = -3C \rightarrow C=0$$

$$-\frac{1}{6} \int \frac{1}{x+2} dx + \frac{1}{6} \int \frac{1}{x-2} + \frac{1}{3} \int \frac{1}{x^2+2} dx$$

$$u=x \quad a=\sqrt{2}$$

$$du=dx$$

$$-\frac{1}{6} \ln|x+2| + \frac{1}{6} \ln|x-2| + \frac{1}{3} \int \frac{1}{u^2+a^2} du$$

$$\frac{1}{6} \ln \left| \frac{x-2}{x+2} \right| + \frac{1}{3} \left(\frac{1}{\sqrt{2}} \arctan \left(\frac{x}{\sqrt{2}} \right) \right) + C$$