8.8 Improper Integrals: Infinite Discontinuities

Pg. 575 #'s 33-36, 39

33) $\int_0^1 \frac{1}{x^2}\,dx$

$\lim_{a\to 0^+} \int_a^1 \frac{1}{x^2}\,dx$

$\lim_{a\to 0^+} \left[\frac{-1}{x}\right]_a^1$

$\lim_{a\to 0^+} \left[-1 - \frac{1}{a}\right]$

$-1 - \frac{1}{0^+}$

$-1 - \infty = -\infty$

Diverges

34) $\int_0^5 \frac{10}{x}\,dx$

$10\lim_{a\to 0^+} \int_a^5 \frac{1}{x}\,dx$

$10\lim_{a\to 0^+} \left[\ln|x|\right]_a^5$

$10\lim_{a\to 0^+} \left[\ln|5| - \ln|a|\right]$

$10\left[\ln|5| - \ln|0^+|\right]$

$10\ln|5| - 10(-\infty)$

∞

Diverges

35) $\int_0^2 \frac{1}{\sqrt[3]{x-1}}$

$\lim_{a\to 1^-} \int_0^a (x-1)^{-1/3}\,dx + \lim_{b\to 1^+} \int_b^2 (x-1)^{-1/3}\,dx$

$\lim_{a\to 1^-} \left[\frac{3}{2}(x-1)^{2/3}\right]_0^a + \lim_{b\to 1^+} \left[\frac{3}{2}(x-1)^{2/3}\right]_b^2$

$\lim_{a\to 1^-} \left[\frac{3}{2}(a-1)^{2/3} - \frac{3}{2}(0-1)^{2/3}\right] + \lim_{b\to 1^+} \left[\frac{3}{2}(2-1)^{2/3} - \frac{3}{2}(b-1)^{2/3}\right]$

$\frac{3}{2}(0^-)^{2/3} - \frac{3}{2}(-1)^{2/3} + \frac{3}{2}(1)^{2/3} - \frac{3}{2}(0^+)^{2/3}$

$0 - \frac{3}{2} + \frac{3}{2} - 0$

0, the integral converges to 0

39) $\int_0^{\pi/2} \sec\theta\,d\theta$

$\lim_{a\to \frac{\pi}{2}^-} \int_0^a \sec\theta\,d\theta$

$\lim_{a\to \frac{\pi}{2}^-} \left[\ln|\sec\theta + \tan\theta|\right]_0^a$

$\lim_{a\to \frac{\pi}{2}^-} \left[\ln|\sec a + \tan a| - \ln|\sec 0 + \tan 0|\right]$

$\ln|\sec\frac{\pi}{2}^- + \tan\frac{\pi}{2}^-| - \ln|\sec 0 + \tan 0|$

$\ln|\infty + \infty| - \ln|1 + 0|$

$\infty - 0$

∞

Diverges

36) $\int_0^8 \frac{3}{\sqrt{8-x}}\,dx$

$\lim_{a\to 8^-} \int_0^a 3(8-x)^{-1/2}\,dx$

$3\lim_{a\to 8^-}\left[-2(8-x)^{1/2}\right]_0^a$

$-6\lim_{a\to 8^-}\left[(8-a)^{1/2}-(8-0)^{1/2}\right]$

$-6\left[\sqrt{0^+}-\sqrt{8}\right]$

$6\sqrt{8}$

$12\sqrt{2}$

The integral converges to $12\sqrt{2}$

39) $\int_0^{\pi/2} \tan\theta\,d\theta$

$\lim_{a\to \frac{\pi}{2}^+} \int_0^a \tan\theta\,d\theta$

$\lim_{a\to \frac{\pi}{2}^+}\left[-\ln|\cos\theta|\right]_0^a$

$\lim_{a\to \frac{\pi}{2}^+}\left[-\ln|\cos(a)|-\ln|\cos(0)|\right]$

$-\ln|\cos(\frac{\pi}{2}^+)|+\ln|1|$

$-\ln|0^+|+\ln|1|$

$-(-\infty)+0$

∞

Diverges