9.10 Taylor and Maclaurin Series

Pg. 673 #'s 1–7

1) $f(x) = e^{2x}$ $\quad f(0) = 1$

$f'(x) = 2e^{2x}$ $\quad f'(0) = 2$

$f''(x) = 4e^{2x}$ $\quad f''(0) = 4$

$f'''(x) = 8e^{2x}$ $\quad f'''(0) = 8$

$$P(x) = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \frac{16x^4}{4!} + \cdots$$

$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!} = \frac{(2x)^n}{n!}$$

2) $f(x) = e^{-4x}$ $\quad f(0) = 1$

$f'(x) = -4e^{-4x}$ $\quad f'(0) = -4$

$f''(x) = 16e^{-4x}$ $\quad f''(0) = 16$

$f'''(x) = -64e^{-4x}$ $\quad f'''(0) = -64$

$$P(x) = 1 - 4x + \frac{16x^2}{2!} - \frac{64x^3}{3!} + \cdots$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^n}{n!} = \frac{(-1)^n (4x)^n}{n!}$$

3) $f(x) = \cos x$ $\quad f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f'(x) = -\sin x$ $\quad f'(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$f''(x) = -\cos x$ $\quad f''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$f'''(x) = \sin x$ $\quad f'''(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$$P(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2 \cdot 2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{2 \cdot 3!}\left(x - \frac{\pi}{4}\right)^3 + \cdots$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\frac{n^2+n}{2}} \sqrt{2}}{2 \cdot n!}\left(x - \frac{\pi}{4}\right)^n$$

4) $f(x) = \sin x$ $\quad f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f'(x) = \cos x$ $\quad f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f''(x) = -\sin x$ $\quad f''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$f'''(x) = -\cos x$ $\quad f'''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$$P(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2 \cdot 2!}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{2 \cdot 3!}\left(x - \frac{\pi}{4}\right)^3 + \cdots$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\frac{n^2-n}{2}} \sqrt{2}}{2 \cdot n!}\left(x - \frac{\pi}{4}\right)^n$$

5) $f(x)=\frac{1}{x}$ $\quad f(1)=1$

$f'(x)=-\frac{1}{x^2}$ $\quad f'(1)=-1$

$f''(x)=\frac{2}{x^3}$ $\quad f''(1)=2$

$f'''(x)=\frac{-6}{x^4}$ $\quad f'''(1)=-6$

$f''''(x)=\frac{-24}{x^5}$ $\quad f''''(1)=24$

$$P(x)=1-1(x-1)+\frac{2(x-1)^2}{2!}-\frac{6(x-1)^3}{3!}+\frac{24(x-1)^4}{4!}-\dots$$

$$P(x)=1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-\dots$$

$$\sum_{n=0}^{\infty}(-1)^n(x-1)^n$$

6) $f(x)=\frac{1}{1-x}$ $\quad f(2)=-1$

$f'(x)=-1(1-x)^{-2}(-1)$

$=\frac{1}{(1-x)^2}$ $\quad f'(2)=1$

$f''(x)=-2(1-x)^{-3}(-1)$

$=\frac{2}{(1-x)^3}$ $\quad f''(2)=-2$

$f'''(x)=\frac{6}{(1-x)^4}$ $\quad f'''(2)=6$

$f''''(x)=\frac{24}{(1-x)^5}$ $\quad f''''(2)=-24$

$$P(x)=-1+(x-2)-\frac{2(x-2)^2}{2!}+\frac{6(x-2)^3}{3!}-\frac{24(x-2)^4}{4!}+\dots$$

$$P(x)=-1+(x-2)-(x-2)^2+(x-2)^3-(x-2)^4+\dots$$

$$\sum_{n=0}^{\infty}(-1)^{n+1}(x-2)^n$$

7) $f(x)=\ln x$ $\quad f(1)=0$

$f'(x)=\frac{1}{x}$ $\quad f'(1)=1$

$f''(x)=\frac{-1}{x^2}$ $\quad f''(1)=-1$

$f'''(x)=\frac{2}{x^3}$ $\quad f'''(1)=2$

$f''''(x)=\frac{-6}{x^4}$ $\quad f''''(1)=-6$

$$P(x)=0+(x-1)-\frac{(x-1)^2}{2!}+\frac{2(x-1)^3}{3!}-\frac{6(x-1)^4}{4!}+\dots$$

$$P(x)=(x-1)-\frac{(x-1)}{2!}+\frac{2(x-1)^3}{3!}-\frac{6(x-1)^4}{4!}+\dots$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{n+1}n!}{(n+1)!}=\frac{(-1)^n(x-1)^{n+1}}{n+1}$$

$$P(x)=(x-1)-\frac{x-1}{2}+\frac{(x-1)^2}{3}-\frac{(x-1)^3}{4}+\dots$$