9.10 Taylor/Maclaurin Manipulation

Pg. 673 #'s 27-35 odd, 47, 59, 65

27) $f(x) = e^{x^2/2}$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{x^2/2} = 1 + \left(\frac{x^2}{2}\right) + \frac{(x^2/2)^2}{2} + \frac{(x^2/2)^3}{3!} + \dots \sum_{n=0}^{\infty} \frac{(x^2/2)^n}{n!}$$

$$e^{x^2/2} = 1 + \frac{x^2}{2} + \frac{x^4}{4 \cdot 2} + \frac{x^6}{8 \cdot 3!} + \dots \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n \cdot n!}$$

29) $f(x) = \ln(1+x)$

$$\ln(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \dots, \sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n+1}$$

$$\ln(1+x) = (1+x-1) - \frac{(1+x-1)^2}{2} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n (1+x-1)^{n+1}}{n+1}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

31) $g(x) = \sin(3x)$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin(3x) = 3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \frac{(3x)^7}{7!} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!}$$

33) $f(x) = \cos(4x)$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(4x) = 1 - \frac{(4x)^2}{2} + \frac{(4x)^4}{4!} - \frac{(4x)^6}{6!} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n (4x)^{2n}}{(2n)!}$$

35) $f(x) = \cos x^{3/2}$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos x^{3/2} = 1 - \frac{(x^{3/2})^2}{2} + \frac{(x^{3/2})^4}{4!} - \frac{(x^{3/2})^6}{6!} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n (x^{3/2})^{2n}}{(2n)!}$$

$$\cos x^{3/2} = 1 - \frac{x^3}{2} + \frac{x^6}{4!} - \frac{x^9}{6!} + \dots \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{(2n)!}$$

47) $f(x) = e^x \sin x$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$x + x^2 + \frac{x^3}{2} + \frac{x^4}{6} + \frac{x^5}{24}$$

$$- \frac{x^3}{6} - \frac{x^4}{6} - \frac{x^5}{12} - \frac{x^6}{36}$$

$$+ \quad \frac{x^5}{120} + \frac{x^6}{240} + \dots$$

$$P(x) = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30}$$

59) $\lim_{x\to 0} \frac{1-\cos x}{x}$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!}$

$\lim_{x\to 0} \frac{1-(1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!})}{x}$

$\lim_{x\to 0} \frac{\frac{x^2}{2}-\frac{x^4}{4!}+\frac{x^6}{6!}}{x}$

$\lim_{x\to 0} \frac{x}{2}-\frac{x^3}{4!}+\frac{x^5}{6!} = \frac{0}{2}-\frac{0}{4!}+\frac{0}{6!} = 0$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

65) $\int_0^1 \frac{\sin x}{x} dx$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

$\int_0^1 \frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}}{x} dx$

$\int_0^1 (1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}) dx$

$\left[x-\frac{x^3}{3\cdot 3!}+\frac{x^5}{5\cdot 5!}-\frac{x^7}{7\cdot 7!}\right]_0^1$

$\underbrace{(1-\frac{1}{18}+\frac{1}{600}}_{\text{solution}} - \underbrace{\frac{1}{35280}}_{\uparrow\ \text{error}}) - (0-0+0-0)$

$\frac{1}{35280} < .0001$

$\boxed{\frac{1703}{1800}}$