CALCULUS BC
WORKSHEET 2 ON TAYLOR POLYNOMIALS

Work the following on **notebook paper**. Use your calculator only on problem 7. Show all work.

1. Find a fifth–degree Maclaurin polynomial for $f(x) = \sin(3x)$.

2. Find a fifth–degree Taylor polynomial for $f(x) = \ln(x-1)$ centered at $x = 2$.

3. Find a fourth–degree Taylor polynomial for $f(x) = e^{(x-4)}$ centered at $x = 4$.

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4. Suppose the function $f(x)$ is approximated near $x = 0$ by a third-degree Taylor polynomial $P_3(x) = -3 + 7x^2 - 2x^3$. Give the value of:

(a) Give the value of: $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$.

(b) Does f have a local maximum, a local minimum, or neither at $x = 0$? Justify your answer.

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5. (a) Find a fourth-degree Maclaurin approximation for $f(x) = \cos x$.

(b) Use your answer to (a) to find $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$.

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6. (a) Find a third-degree Maclaurin approximation for $f(x) = \dfrac{1}{1-2x}$. .

(b) Use your answer to (a) to find $\lim\limits_{x \to 0} \dfrac{f(x) - 1}{x}$.

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7. (a) Find a seventh-degree Maclaurin approximation for $f(x) = \sin x$.

(b) Use your answer to (a) to approximate the value of $\int_0^1 \dfrac{\sin t}{t}\,dt$ so that the error in your approximation is less than $\dfrac{1}{500}$. Justify your answer.

(c) Use your calculator to find the actual value of $\int_0^1 \dfrac{\sin t}{t}\,dt$. What is the error in the approximation you found in (b)?

Answers to Worksheet 2 on Taylor Polynomials

1. $3x - \frac{27x^3}{3!} + \frac{243x^5}{5!}$

2. $(x-2) - \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3} - \frac{(x-2)^4}{4} + \frac{(x-2)^5}{5}$

3. $1 + (x-4) + \frac{(x-4)^2}{2!} + \frac{(x-4)^3}{3!} + \frac{(x-4)^4}{4!}$

4. (a) $f(0) = -3$, $f'(0) = 0$, $f''(0) = 7 \cdot 2!$ or 14, $f'''(0) = -2 \cdot 3!$ or -6

   (b) Since $f'(0) = 0$ and $f''(0)$ is positive, f has a local minimum at $x = 0$ by the Second Derivative Test.

5. (a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ (b) $\frac{1}{2}$

6. (a) $1 + 2x + 4x^2 + 8x^3$ (b) 2

7. (a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ (c) 0.946, 0.002

   (b) $\frac{17}{18}$. Since the terms of the series are alternating, decreasing in magnitude, and having a limit of 0, the $|\text{Error}| < \frac{1}{600} < \frac{1}{500}$