9.1 Sequences

Pg. 592 #'s 1-7 odd, 18, 19, 29-49 odd

1) $a_n = 3^n$

3, 9, 27, 81, 243

3) $a_n = \sin\left(\frac{n\pi}{2}\right)$

1, 0, -1, 0, 1

5) $a_n = (-1)^{n+1}\left(\frac{2}{n}\right)$

$2, -1, \frac{2}{3}, \frac{-1}{2}, \frac{2}{5}$

7) $a_1 = 3 \quad a_{k+1} = 2(a_k - 1)$

3, 4, 6, 10, 18

18) $\frac{n!}{(n+2)!}$

$\frac{n!}{(n+2)(n+1)n!}$

$\frac{1}{(n+2)(n+1)}$

19) $\frac{(2n-1)!}{(2n+1)!}$

$\frac{(2n-1)!}{(2n+1)(2n)(2n-1)!}$

$\frac{1}{(2n+1)(2n)}$

29) $a_n = \frac{5}{n+2}$

$\lim_{n\to\infty} \frac{5}{n+2} = \frac{5}{\infty} = 0$

the sequence converges to zero

31) $a_n = (-1)^n\left(\frac{n}{n+1}\right)$

l'Hop $\lim_{n\to\infty} \frac{n}{n+1} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{1}{1} = 1$

Oscillates between -1 and 1, diverges

33) $a_n = \frac{10n^2+3n+7}{2n^2-6}$

l'Hop $\lim_{n\to\infty} \frac{10n^2+3n+7}{2n^2-6} = \frac{\infty}{\infty}$

l'Hop $\lim_{n\to\infty} \frac{20n+3}{4n} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{20}{4} = 5$

the sequence converges to 5

35) $a_n = \frac{\ln(n^3)}{2n}$

l'Hop $\lim_{n\to\infty} \frac{\ln(n^3)}{2n} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{\frac{1}{n^3}(3n^2)}{2} = \frac{\frac{3}{n}}{2} = \frac{3}{2n} = \frac{3}{\infty} = 0$

the sequence converges to zero

37) $a_n = \frac{(n+1)!}{n!}$

$\lim_{n\to\infty} \frac{(n+1)!}{n!} = \frac{(n+1)n!}{n!} = n+1 = \infty$

the sequence diverges

39) $a_n = \frac{n^p}{e^n}$

$\lim_{n\to\infty} \frac{n^p}{e^n} = \frac{\infty}{\infty}$ (l'Hop)

$\lim_{n\to\infty} \frac{p\,n^{p-1}}{e^n} = \frac{\infty}{\infty}$ (l'Hop)

$\lim_{n\to\infty} \frac{p(p-1)n^{p-2}}{e^n} = \frac{\infty}{\infty}$ (l'Hop)

.....

$\lim_{n\to\infty} \frac{p(p-1)(p-2)\cdots(3)(2)(1)n^{p-p}}{e^n}$

$\lim_{n\to\infty} \frac{p!\,n^0}{e^n} = \frac{p!}{e^n} = \frac{\text{finite}}{\text{infinite}} = 0$

the sequence converges to zero

41) $a_n = 2^{1/n}$

$\lim_{n\to\infty} 2^{1/n} = 2^{1/\infty} = 2^0 = 1$

the sequence converges to 1

43) $a_n = \frac{\sin(n)}{n}$

$\lim_{n\to\infty} \frac{\sin(n)}{n} = \frac{\#\text{ between } -1 \text{ and } 1}{\infty} = 0$

the sequence converges to zero

45) 2, 8, 14, 20

$a_n = 6(n-1) + 2$

47) $-2, 1, 6, 13, 22$

$a_n = n^2 - 3$

49) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

$a_n = \frac{n+1}{n+2}$