Calculus Section 9.2 Geometric Series

-Use properties of infinite geometric series

Homework: page 601 #'s 7, 8, 15, 16, 25, 26, 29, 30, 33, 91-94

Geometric Series

Series of the form $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \ldots + ar^n + \ldots, a \neq 0$ are called **geometric series** with ratio r.

If 0 < |r| < 1, then the geometric series converges. If |r| ≥ 1, then the series diverges.

Example) Determine convergence or divergence of the series.

1) $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

$r = \left|\frac{3}{2}\right| > 1$

the series diverges by the Geometric test

2) $\sum_{n=0}^{\infty} \frac{3}{2^n} = 3\left(\frac{1}{2}\right)^n$

$r = \left|\frac{1}{2}\right| < 1$

the series converges by the Geometric test

3) $\sum_{n=0}^{\infty} \frac{2-3^n}{5^n}$

$\sum_{n=0}^{\infty} 2\left(\frac{1}{5}\right)^n - \sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n$

$r = \left|\frac{1}{5}\right| < 1$ $\quad r = \left|\frac{3}{5}\right| < 1$

both series converge, so the original converges by the Geometric test

4) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^{-n}$

$\sum_{n=0}^{\infty} \left(\frac{\pi}{e}\right)^n$

$r = \left|\frac{\pi}{e}\right| > 1$

the series diverges by the Geometric test

5) $\sum_{n=0}^{\infty} \left(1 + \frac{k}{n}\right)^n$

$r = \left|1 + \frac{k}{n}\right|$

this series will converge if $k < 0$ and diverge if $k > 0$

The Geometric Series is one of the few series where we actually find the sum instead of just saying it converges or diverges.

The sum of a Geometric series $\sum_{n=0}^{\infty} ar^n$ is given by $S = \frac{a}{1-r}$, provided the series converges.

Notice that this summation starts at n = 0. Starting values other than n = 0 will impact how we evaluate the sum.

Example) Find the sum of the following geometric series.

1) $\sum_{n=0}^{\infty} \frac{3}{2^n}$

$r = \frac{1}{2} \quad a = 3$

$\frac{3}{1-\frac{1}{2}} = \frac{3}{1/2} = 6$

the series converges to 6

2) $\sum_{n=0}^{\infty} \frac{2-3^n}{5^n}$

$\sum_{n=0}^{\infty} 2\left(\frac{1}{5}\right)^n - \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n$

$\frac{2}{1-1/5} - \frac{1}{1-3/5}$

$\frac{2}{4/5} - \frac{1}{2/5}$

$\frac{5}{2} - \frac{5}{2} = 0$

the series converges to 0

3) $\sum_{n=1}^{\infty} \left(\frac{-1}{3}\right)^n$

$r = -\frac{1}{3} \quad a = 1$

$\frac{1}{1 - -1/3} = \frac{1}{4/3} = \frac{3}{4}$

$a_0 = 1$

$\frac{3}{4} - 1 = \frac{-1}{4}$

$\sum_{n=1}^{\infty} \left(\frac{-1}{3}\right)^n$ converges to $\frac{-1}{4}$

4) $\sum_{n=2}^{\infty} \frac{-3}{4^n}$

$r = \frac{1}{4} \quad a = -3$

$\frac{-3}{1-1/4} = \frac{-3}{3/4} = -4$

$a_0 = -3 \quad a_1 = \frac{-3}{4}$

$-4 - \left(-3 - \frac{3}{4}\right) = -4 + 3 + \frac{3}{4} = \frac{-1}{4}$

$\sum_{n=2}^{\infty} \frac{-3}{4^n}$ converges to $\frac{-1}{4}$