9.2 Series and the n^{th} Term Test for Divergence

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5) $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}} = \frac{3}{1} + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$

$S_1 = 3$
$S_2 = 4.5$
$S_3 = 5.25$
$S_4 = 5.625$
$S_5 = 5.8125$

6) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} = \frac{1}{1} - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120}$

$S_1 = 1$
$S_2 = 1/2$
$S_3 = 2/3$
$S_4 = .625 = 5/8$
$S_5 = .6\bar{3} = 19/30$

7) $\sum_{n=0}^{\infty} \left(\frac{7}{6}\right)^n$

$\lim_{n\to\infty} \left(\frac{7}{6}\right)^n = \infty \neq 0$

Diverges by n^{th} term test

8) $\sum_{n=0}^{\infty} 4(-1.05)^n$

$4\lim_{n\to\infty} (-1.05)^n = \pm\infty \neq 0$

Diverges by n^{th} term test

9) $\sum_{n=1}^{\infty} \frac{n}{n+1}$

$\lim_{n\to\infty} \frac{n}{n+1} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{1}{1} = 1 \neq 0$

Diverges by n^{th} term test

10) $\sum_{n=1}^{\infty} \frac{n}{2n+3}$

$\lim_{n\to\infty} \frac{n}{2n+3} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{1}{2} = \frac{1}{2} \neq 0$

Diverges by n^{th} term test

11) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

$\lim_{n\to\infty} \frac{n^2}{n^2+1} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{2n}{2n} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{2}{2} = 1 \neq 0$

Diverges by n^{th} term test

12) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{1}{2n(n^2+1)^{-1/2}} = \frac{\sqrt{n^2+1}}{2n} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{2n(n^2+1)^{-1/2}}{2} = \frac{n}{\sqrt{n^2+1}} = \frac{\infty}{\infty}$

l'Hopital's Rule will not work

$$\frac{n}{\sqrt{n^2+1}} \cdot \left(\frac{1/\sqrt{n^2}}{1/\sqrt{n^2}}\right) \to \frac{n/\sqrt{n^2}}{\sqrt{n^2+1}\,(1/\sqrt{n^2})} \to \frac{\frac{n}{n}}{\sqrt{(n^2+1)\left(\frac{1}{n^2}\right)}} \to \frac{1}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}} \to \frac{1}{\sqrt{1+\frac{1}{n^2}}}$$

Diverges by the n^{th} term test

$$\lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = \frac{1}{\sqrt{1+0}} = \frac{1}{\sqrt{1}} = 1 \neq 0$$

13) $\sum_{n=1}^{\infty} \frac{2^n+1}{2^{n+1}}$

$$\frac{2^n+1}{2^{n+1}} \rightarrow \frac{2^n+1}{2\cdot 2^n} \rightarrow \frac{2^n}{2\cdot 2^n} + \frac{1}{2\cdot 2^n} \rightarrow \frac{1}{2} + \frac{1}{2\cdot 2^n}$$

$\lim_{n\to\infty}\left(\frac{1}{2} + \frac{1}{2\cdot 2^n}\right) = \frac{1}{2} + 0 = \frac{1}{2} \neq 0$ Diverges by n^{th} term test

14) $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

$\lim_{n\to\infty} \frac{n!}{2^n} = \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdots}{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdots} = \infty \neq 0$ Diverges by n^{th} term test

56) $\lim_{n\to\infty} a_n = 5$ means that the n^{th} term of the sequence $\{a_n\}$ is approaching 5.

$\sum_{n=1}^{\infty} a_n = 5$ means that the sum of all of the terms of the sequence adds up to 5