Calculus Section 9.2 Series and the nth Term Test for Divergence

-Understand the definition of a convergent infinite series
-Use properties of infinite geometric series
-Use the nth-Term Test for Divergence of an infinite series

Homework: page 601 #'s 5 – 14, 56

One important application of infinite sequences is in representing infinite summations. If $\{a_n\}$ is an infinite sequence, then $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ \ldots\ + a_n + \ \ldots$ is an **infinite series** (or simply, a **series**). The numbers a_1, a_2, a_3, are the **terms** of the series. Unlike sequences which always start at 1, sometimes it is convenient to begin the index at n = 0 (or some other integer).

An infinite series adds up all of the terms of the sequence while a **partial sum** only adds a certain number.

$S_1 = a_1 \qquad S_2 = a_1 + a_2 \qquad S_3 = a_1 + a_2 + a_3 \qquad S_n = a_1 + a_2 + a_3 + \ldots + a_n$

Definitions of Convergent and Divergent Series

For the infinite series $\sum_{n=1}^{\infty} a_n$ the **nth partial sum** is given by $S_n = a_1 + a_2 + a_3 + \ldots + a_n$. If the nth partial sum converges to S, then the series $\sum_{n=1}^{\infty} a_n$ **converges.** The limit S is called the **sum of the series.** If $\{S_n\}$ diverges, then the series **diverges.**

Example) Determine the 4th partial sum for each series.

1) $\sum_{n=1}^{\infty} \frac{1}{2^n}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

$S_4 = \frac{15}{16}$

2) $\sum_{n=1}^{\infty} 1$

$1 + 1 + 1 + 1$

$S_4 = 4$

3) $\sum_{n=3}^{\infty} (-1)^n \frac{1}{n}$

$-\frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6}$

$S_4 = \frac{-7}{60}$

Properties of Infinite Series

If $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$ and c is a real number, then the following series converge to the indicated sums:

1) $\sum_{n=1}^{\infty} ca_n = cA$ 2) $\sum_{n=1}^{\infty} (a_n + b_n) = A + B$ 3) $\sum_{n=1}^{\infty} (a_n - b_n) = A - B$

The next few sections of this chapter are concerned with determining whether a series converges or diverges. There are several tests for convergence/divergence that you will have to know. The first of which is:

nth-Term Test for Divergence

If $\lim\limits_{n\to\infty} a_n \neq 0$, then $\sum\limits_{n=1}^{\infty} a_n$ diverges. *The converse of this ***is not*** true. If $\lim\limits_{n\to\infty} a_n = 0$ the series ***could*** converge*

Example) Determine if the Series Diverges

1) $\sum\limits_{n=0}^{\infty} 2^n$

$\lim\limits_{n\to\infty} 2^n = \infty \neq 0$

Diverges by n^{th} term test

2) $\sum\limits_{n=1}^{\infty} \dfrac{n!}{2n!+1}$

$\lim\limits_{n\to\infty} \dfrac{n!}{2(n!)+1} = \dfrac{1}{2} \neq 0$

Diverges by n^{th} term test

3) $\sum\limits_{n=1}^{\infty} \dfrac{1}{n}$

$\lim\limits_{n\to\infty} \dfrac{1}{n} = 0$

Inconclusive, the series might converge and might diverge