9.3 Integral Test

Pg. 609 #'s 1-9 odd, 23-27 odd

1) $\sum_{n=1}^{\infty} \frac{1}{n+3}$

$\int_1^{\infty} \frac{1}{x+3}\,dx$

$\lim_{a\to\infty} \int_1^a \frac{1}{x+3}\,dx$

$\lim_{a\to\infty} \left[\ln|x+3|\right]_1^a$

$\lim_{a\to\infty} \left[\ln|a+3| - \ln|4|\right]$

$\ln|\infty| - \ln|4|$

∞

The series diverges by the integral test

3) $\sum_{n=1}^{\infty} \frac{1}{2^n}$

$\int_1^{\infty} \left(\frac{1}{2}\right)^x dx$

$\lim_{a\to\infty} \int_1^a \left(\frac{1}{2}\right)^x dx$

$\lim_{a\to\infty} \left[\frac{1}{\ln(1/2)}\left(\frac{1}{2}\right)^u\right]_1^a$

$\lim_{a\to\infty} \left[\frac{1}{\ln(1/2)}\left(\frac{1}{2}\right)^a - \frac{1}{\ln(1/2)}\left(\frac{1}{2}\right)^1\right]$

$0 - \frac{1}{2\ln(1/2)}$

The series converges by the integral test

5) $\sum_{n=1}^{\infty} e^{-n}$

$\int_1^{\infty} \left(\frac{1}{e}\right)^x dx$

$\lim_{a\to\infty} \int_1^a \left(\frac{1}{e}\right)^x dx$

$\lim_{a\to\infty} \left[\frac{1}{\ln(1/e)}\left(\frac{1}{e}\right)^u\right]_1^a$

$\lim_{a\to\infty} \left[-\left(\frac{1}{e}\right)^u - -\left(\frac{1}{e}\right)^1\right]$

$0 + \frac{1}{e}$

The series converges by the integral test

7) $\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26}$

$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

$\lim_{a\to\infty} \int_1^a \frac{1}{x^2+1}\,dx$

$\lim_{a\to\infty} \left[\arctan x\right]_1^a$

$\lim_{a\to\infty} \left[\arctan(a) - \arctan(1)\right]$

$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

The series converges by the integral test

9) $\frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + \frac{\ln 6}{6}$

$\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

$\lim_{a\to\infty} \int_2^a \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\lim_{a\to\infty} \left[\frac{1}{2}(\ln x)^2\right]_2^a$

$\lim_{a\to\infty} \left[\frac{1}{2}(\ln a)^2 - \frac{1}{2}(\ln 2)^2\right]$

$\infty - \frac{1}{2}(\ln 2)^2$

The series diverges by the integral test

23) $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k + c}$

$\lim_{a\to\infty} \int_1^a \frac{x^{k-1}}{x^k + c}\, dx$

$u = x^k + c$

$du = kx^{k-1}\, dx$

$\frac{1}{k} du = x^{k-1}\, dx$

$\lim_{a\to\infty} \left[\frac{1}{k} \ln|x^k + c|\right]_1^a$

$\lim_{a\to\infty} \left[\frac{1}{k} \ln|a^k + c| - \frac{1}{k} \ln|1 + c|\right]$

$\frac{1}{k} \ln|\infty| - \frac{1}{k} \ln|1 + c|$

∞

the series diverges by the integral test

25) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

The integral test does not apply because some terms are negative

27) $\sum_{n=1}^{\infty} \frac{2 + \sin(n)}{n}$

The integral test does not apply because the terms are not strictly decreasing.