Calculus Section 9.3 Integral Test

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#'s 1 – 9 odd, 23 – 27 odd

-Use the Integral Test to determine whether an infinite series converges or diverges.

The Integral Test
If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)dx$$

either both converge or both diverge.

Examples) Use the Integral test to determine convergence or divergence of each series.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

$$\lim_{a\to\infty} \int_1^a \frac{x}{x^2+1}dx \qquad u = x^2+1, \quad du = 2x\,dx, \quad \tfrac{1}{2}du = x\,dx$$

$$\lim_{a\to\infty} \frac{1}{2}\left[\ln(x^2+1)\right]_1^a$$

$$\lim_{a\to\infty} \left[\frac{1}{2}\ln(a^2+1) - \frac{1}{2}\ln(1+1)\right]$$

$$\frac{1}{2}\ln(\infty) - \frac{1}{2}\ln(2) = \infty$$

The series diverges by the integral test because $\int_1^{\infty} \frac{x}{x^2+1}dx$ diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

$$\lim_{a\to\infty} \int_1^a \frac{1}{x^2+1}dx$$

$$\lim_{a\to\infty} \left[\arctan(x)\right]_1^a$$

$$\lim_{a\to\infty} \left[\arctan(a) - \arctan(1)\right]$$

$$\arctan(\infty) - \arctan(1)$$

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

The series converges by the integral test because $\int_1^{\infty} \frac{1}{x^2+1}dx$ converges.

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

$$\lim_{a\to\infty} \int_2^a \frac{1}{x\ln x}dx \qquad u = \ln x, \quad du = \tfrac{1}{x}dx$$

$$\lim_{a\to\infty} \left[\ln|\ln x|\right]_2^a$$

$$\lim_{a\to\infty} \left[\ln|\ln a| - \ln|\ln 2|\right]$$

$$\infty - \ln|\ln 2| = \infty$$

The series diverges by the integral test because $\int_2^{\infty} \frac{1}{x\ln x}dx$ diverges.