9.3 p-Series Test

Pg. 609 #'s 31-38

31) $\sum\limits_{n=1}^{\infty} \frac{1}{n^{1/4}}$

$f(x) = x^{-1/4}$

$\lim\limits_{a \to \infty} \int_{1}^{a} x^{-1/4} dx$

$\lim\limits_{a \to \infty} \left[\frac{4}{3} x^{3/4} \right]_{1}^{a}$

$\lim\limits_{a \to \infty} \left[\frac{4}{3} a^{3/4} - \frac{4}{3}(1)^{3/4} \right]$

$\infty - \frac{4}{3} = \infty$

The series diverges by the integral test because $\int_{1}^{\infty} \frac{1}{x^{1/4}} dx$ diverges.

32) $\sum\limits_{n=1}^{\infty} \frac{1}{n^5}$

$f(x) = x^{-5}$

$\lim\limits_{a \to \infty} \int_{1}^{a} x^{-5} dx$

$\lim\limits_{a \to \infty} \left[-\frac{1}{4} x^{-4} \right]_{1}^{a}$

$\lim\limits_{a \to \infty} \left[\frac{-1}{4a^4} - \frac{-1}{4(1)^4} \right]$

$\frac{-1}{\infty} + \frac{1}{4} = \frac{1}{4}$

The series converges by the integral test because $\int_{1}^{\infty} \frac{1}{x^5} dx$ diverges.

33) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

$p = \frac{1}{5} \leq 1$

The series diverges by the p-series test

34) $\sum\limits_{n=1}^{\infty} \frac{3}{n^{5/3}}$

$p = \frac{5}{3} > 1$

The series converges by the p-series test

35) $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots$

$\sum\limits_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \frac{1}{n^{3/2}}$

$p = \frac{3}{2} > 1$

The series converges by the p-series test

36) $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \cdots$

$\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}} = \frac{1}{n^{2/3}}$

$p = \frac{2}{3} \leq 1$

The series diverges by the p-series test

37) $\sum\limits_{n=1}^{\infty} \frac{1}{n^{1.04}}$

$p = 1.04 > 1$

The series converges by the p-series test

38) $\sum\limits_{n=1}^{\infty} \frac{1}{n^{\pi}}$

$p = \pi > 1$

The series converges by the p-series test