Calculus Section 9.3 p-Series Test

Homework: page 609 #'s 31 – 38

-Use properties of the p-series and harmonic series

Definition of a p-Series

A p-series is a type of series that follows the following pattern:

$$\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots$$

where p is a positive constant. For p = 1, the series $\sum_{n=1}^{\infty}\frac{1}{n}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots$ is called the <u>**harmonic series**</u>.

Convergence of p-Series

The p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots$ (The proof follows from the Integral test.)

1) converges if p > 1

2) diverges if 0 < p ≤ 1

Proof) Divergence of the Harmonic Series

Show that the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$ diverges.

$$\underbrace{1}_{1}+\underbrace{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}_{\text{\# greater than 1}}+\underbrace{\frac{1}{5}+\frac{1}{6}+\cdots+\frac{1}{12}}_{\text{\# greater than 1}}+\underbrace{\frac{1}{13}+\cdots+\frac{1}{34}}_{\text{\# greater than 1}}+\underbrace{\frac{1}{35}+\cdots+\frac{1}{94}}_{\text{\# greater than 1}}+\underbrace{\frac{1}{95}+\cdots+}_{\text{\# greater than 1}}\ \text{etc.}+\cdots$$

the series diverges (very slowly)

Example) Convergent and Divergent p-Series

1) $\sum_{n=1}^{\infty}\frac{1}{n^2}$

$p=2>1$

the series converges by the p-series test

2) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

$p=\frac{1}{2}\le 1$

the series diverges by the p-series test

3) $\sum_{n=1}^{\infty}\frac{1}{n}$

$p=1\le 1$

the series diverges by the p-series test