9.4 Direct Comparison Test

pg. 616 #'s 3-12, 49-54

3) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ compare to $\frac{1}{2n}$

$\frac{1}{2n} < \frac{1}{2n-1}$

$\int_1^{\infty} \frac{1}{2x}\,dx = \infty$

$\sum_{n=1}^{\infty} \frac{1}{2n}$ diverges by integral test,

so $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges by direct comp.

4) $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ compare to $\frac{1}{n^2}$

$\frac{1}{n^2} > \frac{1}{3n^2+2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series $(p>1)$, so

$\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ converges by direct comparison

5) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$ compare to $\frac{1}{\sqrt{n}}$

$\frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n}-1}$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series $(p \leq 1)$,

so $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$ diverges by direct comp.

6) $\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$ compare to $\frac{4^n}{5^n}$

$\frac{4^n}{5^n} > \frac{4^n}{5^n+3}$

$\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$ converges by Geometric test $(r<1)$,

so $\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$ converges by direct comp.

7) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n+1}$ compare to $\frac{1}{n+1}$

$\frac{1}{n+1} < \frac{\ln(n)}{n+1}$ for $x>e$

$\sum_{n=2}^{\infty} \frac{1}{n+1}$ diverges by the integral test

so $\sum_{n=2}^{\infty} \frac{\ln(n)}{n+1}$ diverges by direct comp.

8) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ compare to $\frac{1}{\sqrt{n^3}}$

$\frac{1}{\sqrt{n^3}} > \frac{1}{\sqrt{n^3+1}}$

$\sum_{n=1}^{\infty} n^{3/2}$ converges by p-series $(p>1)$,

so $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ converges by direct comp.

9) $\sum_{n=0}^{\infty} \frac{1}{n!}$ compare to $\frac{1}{n^2}$

$\frac{1}{n^2} > \frac{1}{n!} \quad n \geq 4$

$\sum_{n=0}^{\infty} \frac{1}{n^2}$ converges by p-series $(p>1)$,

so $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges by direct comp.

10) $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ compare to $\frac{1}{n}$

$\frac{1}{n} < \frac{1}{4\sqrt[3]{n}-1}$ for $n \geq 7$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series $(p \leq 1)$,

so $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ diverges by direct comp.

11) $\sum_{n=0}^{\infty} e^{-n^2}$ compare to $\frac{1}{e^n}$

$\frac{1}{e^n} > \frac{1}{e^{n^2}}$

$\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges by Geometric test $(r<1)$,

so $\sum_{n=0}^{\infty} e^{-n^2}$ converges by direct comp.

12) $\sum_{n=1}^{\infty} \frac{3^n}{2^n-1}$ compare to $\frac{3^n}{2^n}$.

$\frac{3^n}{2^n} < \frac{3^n}{2^n-1}$

$\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$ diverges by Geometric test $(r \geq 1)$,

so $\sum_{n=1}^{\infty} \frac{3^n}{2^n-1}$ diverges by direct comparison.

49) False, a smaller series converging says nothing about a larger series

50) True, provided a_1 through a_9 are finite

51) True

52) False, either b_n or c_n could converge

53) True

54) False, a larger series diverging says nothing about a smaller series