Calculus Section 9.4 Direct Comparison Test

·Use the direct comparison test to determine convergence.

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The convergence tests so far (nth-term, geometric, integral, and p-series) have been fairly simple and the series have special characteristics that make finding convergence easy. Any slight deviation from those characteristics can yield a series where the previous tests would not apply. For example,

1) $\sum_{n=0}^{\infty} \frac{1}{2^n}$ is geometric, but $\sum_{n=0}^{\infty} \frac{n}{2^n}$ is not.

2) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is a p-series, but $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ is not.

3) $a_n = \frac{n}{(n^2+3)^2}$ is easily integrated, but $b_n = \frac{n^2}{(n^2+3)^2}$ is not.

The direct comparison test is a tool that we can use to determine convergence for complicated, positive series by comparing them with simpler series.

Direct Comparison Test

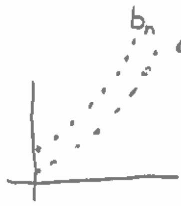
Let $0 < a_n \le b_n$ for all n.

1) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

2) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

a_n (being smaller) cannot exceed b_n

b_n (being larger) cannot sum to a value less than a_n



Example) Determine Convergence or Divergence

1) $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$ compare to $\frac{1}{3^n}$

$$\frac{1}{3^n} > \frac{1}{2+3^n}$$

$\sum_{n=1}^{\infty} \frac{1}{3^n}$ converges by the Geometric test $(r<1)$. Therefore, $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$ also converges by direct comparison.

2) $\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}-2}$ compare to $\frac{1}{\sqrt{n}}$

$$\frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n}-2}$$

$\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}}$ diverges by the p-series test $(p \le 1)$. Therefore, $\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}-2}$ diverges by direct comparison.

3) $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ compare to $\frac{1}{n}$

$\frac{1}{n} < \frac{1}{4\sqrt[3]{n}-1}$ for $n \geq 7$ $\leftarrow$ It is okay to exclude a finte num. of finite terms as they will not affect whether the series converges or diverges.

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test ($p \leq 1$),

so $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$ diverges by direct comparison.