9.5 Alternating Series Remainder and Abs/Cond. Convergence

Pg. 625 #'s 30-32, 37-47 odd, 61, 62

30) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3^n} = \frac{1}{3} - \frac{2}{9} + \frac{3}{27} - \frac{4}{81} + \frac{5}{243} - \frac{6}{729} \,\vdots\, + \frac{7}{2187}$

$S_6 = \frac{5}{27}$ the error is no greater than $\frac{7}{2187}$

31) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$

$\frac{1}{n^3} < \frac{1}{1000}$

$1000 < n^3$

$10 < n$

10 terms

32) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$\frac{1}{n^2} < \frac{1}{1000}$

$1000 < n^2$

$31.6 < n$

31 terms

37) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges by geometric test $(r<1)$, so $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$ converges absolutely.

39) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

$\sum_{n=1}^{\infty} \frac{1}{n!}$ converges by direct comparison $\left(\sum_{n=1}^{\infty} \frac{1}{n^2}\right)$, so $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ converges absolutely.

41) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series test $(p \leq 1)$.

1) $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓

2) $\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$ ✓

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges conditionally

43) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+1)^2}$

$\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2}$ diverges by the n^{th} term test.

1) $\lim_{n\to\infty} \frac{n^2}{(n+1)^2} \neq 0$ X

The series diverges absolutely.

45) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges by the integral test.

1) $\lim_{n\to\infty} \frac{1}{n \ln n} = 0$ ✓

2) $\frac{1}{(n+1)\ln(n+1)} \leq \frac{1}{n \ln n}$ ✓

The series converges conditionally.

47) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3-5}$

$\sum_{n=2}^{\infty} \frac{n}{n^3-5}$ converges by limit comparison (via the p-series test).

$\sum_{n=2}^{\infty} \frac{n}{n^3-5}$ converges absolutely.

61) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \underset{n=1}{-1} + \underset{n=2}{\frac{1}{2}} - \underset{n=3}{\frac{1}{3}} + \underset{n=4}{\frac{1}{4}} \cdots$

$n=100$ positive $\quad n=101$ negative

S_{100} is an overestimate; true

62) True (Would be false if: $\sum a_n$ and $\sum b_n$ diverge, then $\sum a_n b_n$ diverges)