Calculus Section 9.5 Alternating Remainder and Conditional/Absolute Convergence

-Use the Alternating Series Remainder to determine convergence

-Classify convergence as absolute or conditional

Homework: page #625 #'s 30 – 32, 37 – 47 odd, 61, 62

Alternating Series Remainder

If an alternating series converges, then the sum of the series can be approximated by the partial sum S_n. The error associated with this sum is less than or equal to the first neglected term (the n + 1 term).

Remainder = Maximum Error = $|S - S_n| \le a_{n+1}$

Example) Approximating the Sum of an Alternating Series

Use the 4th partial sum to approximate the sum of the series. Determine a reasonable interval for the actual sum of the series. Is the partial sum S_4 an over or underestimate?

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n!} = \frac{1}{1} - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120}$$

$S_4 = \frac{5}{8}$

Sum: $\frac{5}{8} - \frac{1}{120} \le \text{Sum} \le \frac{5}{8} + \frac{1}{120}$

$.61\overline{6} \le \text{Sum} \le .6\overline{3}$

S_4 is an underestimate of the actual sum because the last term was subtracted.

Example) What alternating series partial sum will have an error less than or equal to 0.001.

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}$$

$\frac{1}{2n-1} \le \frac{1}{1000}$

$1000 \le 2n-1$

$1001 \le 2n$

$500.5 \le n$

$\boxed{S_{500}}$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)!}$$

$\frac{1}{(n+1)!} \le \frac{1}{1000}$

$\frac{1}{1} \quad \frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{24} \quad \frac{1}{120} \quad \frac{1}{720} \quad \frac{1}{5040}$

$\boxed{S_5}$

Example) Given $P_n(x) = \sum_{k=1}^{n} (-1)^k \frac{x^k}{k^2+k+1}$. What is the smallest number M for which the alternating error bound guarantees that $|f(1) - P_4(1)| \le M$? error

actual sum partial sum

M will come from the 5th term since $P_4(1)$ is the 4th partial sum.

$x=1$ because the sum is for $f(1) - P_4(1)$; $k=5$.

$$(-1)^5 \frac{1^5}{5^2+5+1} = \frac{-1}{25+5+1} = \boxed{\frac{-1}{31}}$$

Definitions of Absolute and Conditional Convergence

A series **converges absolutely** (is absolutely convergent) if $\sum |a_n|$ converges.

A series **converges conditionally** (is conditionally convergent) if $\sum a_n$ converges but $\sum |a_n|$ diverges.

A conditionally convergent series converges only on the condition that it alternates (classic example: harmonic series) whereas an absolutely convergent series will converge whether it alternates or not.

Example) Does the series converge absolutely, converge conditionally, or diverge?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$$

$$\frac{-1}{1} + \frac{1}{\sqrt[3]{4}} - \frac{1}{\sqrt[3]{9}} + \dots$$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$ diverges by the p-series test ($p \le 1$)

1) $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^2}} = 0$ ✓

2) $\frac{1}{\sqrt[3]{(n+1)^2}} \le \frac{1}{\sqrt[3]{n^2}}$ ✓

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$ converges conditionally

$$\sum_{n=1}^{\infty} \frac{(-1)^{\frac{n^2+n}{2}}}{3^n}$$

$$\frac{-1}{3} - \frac{1}{9} + \frac{1}{27} + \frac{1}{81} - \frac{1}{243} - \frac{1}{729} \dots$$

$\sum_{n=1}^{\infty} \frac{1}{3^n}$ converges by geometric test. So $\sum_{n=1}^{\infty} \frac{(-1)^{\frac{n^2+n}{2}}}{3^n}$ converges absolutely.