9.5 Alternating Series

Pg. 625 #'s 5-21 odd

5) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

1) $\lim_{n\to\infty} \frac{1}{n+1} = 0$ ✓

2) $\frac{1}{n+2} \leq \frac{1}{n+1}$ ✓

The series converges by alternating series test

7) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$

1) $\lim_{n\to\infty} \frac{1}{3^n} = 0$ ✓

2) $\frac{1}{3^{n+1}} \leq \frac{1}{3^n}$ ✓

The series converges by the alt. series test

9) $\sum_{n=1}^{\infty} \frac{(-1)^n(5n-1)}{4n+1}$

1) $\lim_{n\to\infty} \frac{5n-1}{4n+1} = \frac{5}{4}$ X

Diverges by the n^{th} term test.

11) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)}$

1) $\lim_{n\to\infty} \frac{n}{\ln(n+1)} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{1}{1/n+1} = n+1 = \infty$ X

Diverges by n^{th} term test

13) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

1) $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓

2) $\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$ ✓

The series converges by the alternating series test

15) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{\ln(n+1)}$

1) $\lim_{n\to\infty} \frac{n+1}{\ln(n+1)} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{1}{1/n+1} = n+1 = \infty$

The series diverges by n^{th} term test

17) $\sum_{n=1}^{\infty} \sin\left(\frac{(2n-1)\pi}{2}\right)$

1) $\lim_{n\to\infty} 1 = 1$

Diverges by n^{th} term test

$\sin\frac{(2n-1)\pi}{2}$ is the alternating part

19) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

1) $\lim_{n\to\infty} \frac{1}{n!} = 0$ ✓

2) $\frac{1}{(n+1)!} \leq \frac{1}{n!}$ ✓

The series converges by the alternating series test

21) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$

1) $\lim_{n\to\infty} \frac{\sqrt{n}}{n+2} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{\frac{1}{2}n^{-1/2}}{1} = \frac{1}{2\sqrt{n}} = 0$ ✓

2) $\frac{\sqrt{n+1}}{n+3} \leq \frac{\sqrt{n}}{n+2}$ ✓

The series converges by the alternating series test