Calculus Section 9.5 Alternating Series Test

-Use the alternating series test to determine convergence

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Most of the tests that we've used so far have dealt with only positive terms (geometric test withstanding). A series whose terms switch between positive and negative is called an **alternating series**. An alternating series cannot have two terms of the same sign back-to-back.

Alternating Series Test

Let $a_n > 0$. The alternating series:

$$\sum_{n=1}^{\infty}(-1)^n a_n \text{ and } \sum_{n=1}^{\infty}(-1)^{n+1} a_n$$

will converge if the following two conditions are met:

1) $\lim\limits_{n\to\infty} a_n = 0$ and 2) $a_{n+1} \le a_n$ for all n

If the test fails the first condition, then the series diverges by the nth term test.

Example) Using the Alternating Series Test

Determine the convergence or divergence of $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

1) $\lim\limits_{n\to\infty} \frac{1}{n} = 0$ ✓

2) $\frac{1}{n+1} \le \frac{1}{n}$ ✓

The series converges by the alternating series test.

Example) Use the Alternating Series Test

1) $\sum_{n=1}^{\infty}\frac{n}{(-2)^{n-1}} = \frac{n}{(-1)^{n-1}(2)^{n-1}} = (-1)^{n-1}\frac{2n}{2^n}$

1) $\lim\limits_{n\to\infty} \frac{2n}{2^n} = 0$ ✓

2) $\frac{2(n+1)}{2^{n+1}} \le \frac{2n}{2^n}$ ✓

The series converges by the alternating series test.

2) $\sum_{n=1}^{\infty}\frac{1}{n^2}\cos(\pi x)$

$\overbrace{\cos(\pi) \quad \cos(2\pi) \quad \cos(3\pi)}^{\text{alternating part}}$

$-1 \quad +1 \quad -1$

1) $\lim\limits_{n\to\infty} \frac{1}{n^2} = 0$ ✓

2) $\frac{1}{(n+1)^2} \le \frac{1}{n^2}$ ✓

The series converges by the alternating series test.

3) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n}$

1) $\lim_{n \to \infty} \frac{n+1}{n} = 1$

The series diverges by the n^{th} term test.