9.6 All Tests

Pg 634 #'s 51-66

51) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5}{n}$

1) $\lim_{n\to\infty} \frac{5}{n} = 0$ ✓

2) $\frac{5}{n+1} \le \frac{5}{n}$ ✓

Converges by alternating series

52) $\sum_{n=1}^{\infty} \frac{100}{n} = 100\left(\frac{1}{n}\right)$

diverges by p-series

53) $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}} = 3\left(\frac{1}{n^{3/2}}\right)$

converges by p-series

54) $\sum_{n=1}^{\infty} \left(\frac{2\pi}{3}\right)^n$

diverges by Geometric test

55) $\sum_{n=1}^{\infty} \frac{5n}{2n-1}$

$\lim_{n\to\infty} \frac{5n}{2n-1} = \frac{5}{2}$

diverges by n^{th} term

56) $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ compare to $\frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} \cdot \frac{2n^2+1}{n} = \frac{2n^2+1}{n^2} = 2$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series and

$\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ diverges by limit comp.

57) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-2}}{2^n}$

$\lim_{n\to\infty} \frac{3^{n-1}}{2^{n+1}} \cdot \frac{2^n}{3^{n-2}}$

$\lim_{n\to\infty} \frac{3}{2} > 1$

diverges by ratio test

58) $\sum_{n=1}^{\infty} \frac{10}{3\sqrt{n^3}} = \frac{10}{3}\left(\frac{1}{n^{3/2}}\right)$

converges by p-series

59) $\sum_{n=1}^{\infty} \frac{10n+3}{n2^n}$

$\lim_{n\to\infty} \frac{10(n+1)+3}{(n+1)2^{n+1}} \cdot \frac{n2^n}{10n+3}$

$\lim_{n\to\infty} \frac{(10n+10+3)n}{2(n+1)(10n+3)}$

$\lim_{n\to\infty} \frac{10n^2+13n}{20n^2+26n+6} = \frac{1}{2} < 1$

converges by ratio test

60) $\sum_{n=1}^{\infty} \frac{2^n}{4n^2-1}$

$\lim_{n\to\infty} \frac{2^n}{4n^2-1} = \frac{\infty}{\infty} \xrightarrow{\text{l'Hop}} \lim_{n\to\infty} \frac{(\ln 2)2^n}{8n} = \frac{\infty}{\infty} \xrightarrow{\text{l'Hop}} \lim_{n\to\infty} \frac{(\ln 2)(\ln 2)2^n}{8} = \infty$

The series diverges by nth term test

61) $\sum_{n=1}^{\infty} \frac{\cos n}{3^n}$

$\sum_{n=1}^{\infty} \left|\frac{\cos n}{3^n}\right|$ converges absolutely

by direct comparison $\left|\frac{\cos n}{3^n}\right| \le \frac{1}{3^n}$,

so $\sum_{n=1}^{\infty} \frac{\cos n}{3^n}$ will converge.

62) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

1) $\lim_{n\to\infty} \frac{1}{n \ln n} = 0$ ✓

2) $\frac{1}{(n+1)\ln(n+1)} < \frac{1}{n \ln n}$ ✓

converges by alternating series test

63) $\sum_{n=1}^{\infty} \frac{n!}{n 7^n}$

$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)7^{n+1}} \cdot \frac{n7^n}{n!}$

$\lim_{n\to\infty} \frac{(n+1)n!}{(n+1)7^{n+1}} \cdot \frac{n7^n}{n!}$

$\lim_{n\to\infty} \frac{n}{7} = \infty > 0$

diverges by ratio test

64) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

$f(x) = \frac{\ln x}{x^2}$

$\int_1^{\infty} \frac{\ln x}{x^2} dx$ $\quad u = \ln x \quad v = -\frac{1}{x}$

$du = \frac{1}{x} dx \quad dv = \frac{1}{x^2} dx$

$\int_1^{\infty} \frac{\ln x}{x^2} dx = -\frac{1}{x}\ln x - \int_1^{\infty} -\frac{1}{x}\left(\frac{1}{x}\right) dx$

$= -\frac{1}{x}\ln x + \int_1^{\infty} x^{-2} dx$

$= \left[-\frac{1}{x}\ln x - \frac{1}{x}\right]_1^{\infty}$

$\left(\frac{-\ln\infty}{\infty} - \frac{1}{\infty}\right) - \left(\frac{\ln 1}{1} - \frac{1}{1}\right)$

$-\frac{\ln\infty}{\infty} - 0 - 0 + 1$

$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$

L'Hop

$\lim_{x\to\infty} \frac{1/x}{1} = \frac{1}{x} = 0$

$0 - 0 - 0 + 1 = 1$ The series converges by the integral test

65) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-1}}{n!}$

1) $\lim_{n\to\infty} \frac{3^{n-1}}{n!} = 0$ ✓

2) $\frac{3^n}{(n+1)!} \le \frac{3^{n-1}}{n!}$ ✓ $n \ge 2$

The series converges by alternating series test

66) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n 2^n}$

$\lim_{n\to\infty} \sqrt[n]{\frac{3^n}{n 2^n}}$

$\lim_{n\to\infty} \frac{3}{\sqrt[n]{n}\, 2} = \frac{3}{(1)(2)} = \frac{3}{2} > 1$

The series diverges by root test