Calculus Section 9.6 Finding the Right Test

Homework: page 634 #'s 51 - 66

-Use general guidelines to determine which test would work best for a given series

The following table can be found on page 632 of the textbook.

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
nth-Term test for divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\|r\| < 1$	$\|r\| \geq 1$	$Sum = \frac{a}{1-r}$; sum must start at zero.
p-Series	$\sum_{n=1}^{\infty} \frac{1}{p^n}$	$p > 1$	$0 < p \leq 1$	
Integral Test	$\sum_{n=1}^{\infty} a_n$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	f is continuous, positive, and decreasing
Direct Comparison Test	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L$ and $\sum_{n=1}^{\infty} b_n$ diverges	L must be positive and finite (not zero, not infinity)
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^n a_n$	1) $\lim_{n\to\infty} a_n = 0$ and 2) $a_{n+1} \leq a_n$		Remainder: $\|R_n\| \leq a_{N+1}$
Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\|a_n\|} < 1$	$\lim_{n\to\infty} \sqrt[n]{\|a_n\|} > 1$	Inconclusive if $\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = 1$
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| < 1$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| > 1$	Inconclusive if $\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| = 1$