9.6 Ratio Test

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21) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$

$\lim_{n\to\infty} \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3}$

$\lim_{n\to\infty} \frac{(n+1)^3}{3n^3} = \frac{1}{3} < 1$

The series converges by the ratio test.

22) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2)}{n(n+1)}$

$\lim_{n\to\infty} \frac{(n+3)}{(n+1)(n+2)} \cdot \frac{n(n+1)}{(n+2)}$

$\lim_{n\to\infty} \frac{n^2+3n}{n^2+4n+4} = 1$

1) $\lim_{n\to\infty} \frac{n+2}{n(n+1)} = 0$ ✓

2) $\frac{n+3}{(n+1)(n+2)} \leq \frac{n+2}{n(n+1)}$ ✓

The ratio test is inconclusive, but the series converges by the alternating series test.

23) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$

$\lim_{n\to\infty} \frac{2 \cdot n!}{(n+1)!} = \frac{2n!}{(n+1)n!}$

$\lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$

The series converges by the ratio test.

24) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3/2)^n}{n^2}$

$\lim_{n\to\infty} \frac{(3/2)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(3/2)^n}$

$\lim_{n\to\infty} \frac{(3/2)n^2}{n^2+2n+1} = \frac{3}{2} > 1$

The series diverges by the ratio test.

25) $\sum_{n=1}^{\infty} \frac{n!}{n3^n}$

$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)3^{n+1}} \cdot \frac{n3^n}{n!}$

$\lim_{n\to\infty} \frac{(n+1)n!n}{3(n+1)n!} = \frac{n}{3} = \infty > 1$

The series diverges by the ratio test.

26) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

$\lim_{n\to\infty} \frac{(2(n+1))!}{(n+1)^5} \cdot \frac{n^5}{(2n)!}$

$\lim_{n\to\infty} \frac{(2n+2)!}{(n+1)^5} \cdot \frac{n^5}{(2n)!}$

$\lim_{n\to\infty} \frac{(2n+2)(2n+1)(2n)!}{(n+1)^5} \cdot \frac{n^5}{(2n)!}$

$\lim_{n\to\infty} \frac{(2n+2)(2n+1)n^5}{(n+1)^5}$

$\lim_{n\to\infty} \frac{4n^7+6n^6+2n^5}{(n+1)^5} = \infty > 1$

The series diverges by the ratio test.

27) $\sum_{n=0}^{\infty} \frac{e^n}{n!}$

$\lim_{n\to\infty} \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n}$

$\lim_{n\to\infty} \frac{e \cdot n!}{(n+1)n!} = \frac{e}{n+1} = 0 < 1$

The series converges by the Ratio test.

28) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!}$

$\lim_{n\to\infty} \frac{(n+1)n!}{(n+1)(n+1)^n} \cdot \frac{n^n}{n!}$

$\lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \left(\frac{n}{n+1}\right)^n$

$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$

$y = \left(\frac{n}{n+1}\right)^n$

$\ln y = n \ln\left(\frac{n}{n+1}\right)$

$\lim_{n\to\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{1/n} = \frac{0}{0}$

l'Hop

$\lim_{n\to\infty} \frac{\left(\frac{1}{n/n+1}\right)\left(\frac{n(1)-(n+1)(1)}{(n+1)^2}\right)}{-1/n^2}$

$\lim_{n\to\infty} \frac{\frac{n+1}{n}\left(\frac{n-n+1}{(n+1)^2}\right)}{-1/n^2}$

$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)\left(\frac{1}{(n+1)^2}\right)(-n^2)$

$\lim_{n\to\infty} \frac{-n}{n+1} = -1$

$e^{\ln y} = e^{-1}$

$y = 1/e$

$\frac{1}{e} < 1$

The series converges by the ratio test.

29) $\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^n}$

$\lim_{n\to\infty} \frac{6^{n+1}}{(n+2)^{n+1}} \cdot \frac{(n+1)^n}{6^n}$

$\lim_{n\to\infty} \frac{6(n+1)^n}{(n+2)^{n+1}} = 0 < 1$

The series converges by the ratio test.

30) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$

$\lim_{n\to\infty} \frac{((n+1)!)^2}{(3(n+1))!} \cdot \frac{(3n)!}{(n!)^2}$

$\lim_{n\to\infty} \frac{(n+1)^2(n!)^2}{(3n+3)(3n+2)(3n+1)(3n)!} \cdot \frac{(3n)!}{(n!)^2}$

$\lim_{n\to\infty} \frac{(n+1)^2}{(3n+3)(3n+2)(3n+1)} = 0 < 1$

The series converges by the ratio test.

31) $\sum_{n=0}^{\infty} \frac{5^n}{2^n+1}$

$\lim_{n\to\infty} \frac{5^{n+1}}{2^{n+1}+1} \cdot \frac{2^n+1}{5^n}$

$\lim_{n\to\infty} \frac{5(2^n)+5}{2(2^n)+2}$

l'Hop

$\lim_{n\to\infty} \frac{5(\ln 2)2^n}{2(\ln 2)2^n} = \frac{5}{2} > 1$

The series diverges by the ratio test

32) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$

$\lim_{n\to\infty} \frac{2^{4n+4}}{(2n+3)!} \cdot \frac{(2n+1)!}{2^{4n}}$

$\lim_{n\to\infty} \frac{2^4(2n+1)!}{(2n+3)(2n+2)(2n+1)!}$

$\lim_{n\to\infty} \frac{2^4}{(2n+3)(2n+2)} = 0 < 1$

The series converges by the ratio test.