9.6 Root Test

Pg 633 #'s 35-49 odd

35) $\sum_{n=1}^{\infty} \frac{1}{5^n} = \left(\frac{1}{5}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{1}{5}\right)^n}$

$\lim_{n\to\infty} \frac{1}{5} = \frac{1}{5} < 1$

The series converges by the root test

37) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n}$

$\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} < 1$

The series converges by the root test

39) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{n+3}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{3n+2}{n+3}\right)^n}$

$\lim_{n\to\infty} \frac{3n+2}{n+3} = 3 > 1$

The series diverges by the root test.

41) $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

$\lim_{n\to\infty} \sqrt[n]{\frac{1}{(\ln n)^n}}$

$\lim_{n\to\infty} \frac{1}{\ln n} = 0 < 1$

The series converges by the root test

43) $\sum_{n=1}^{\infty} \left(2\sqrt[n]{n}+1\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(2\sqrt[n]{n}+1\right)^n}$

$\lim_{n\to\infty} 2\sqrt[n]{n}+1$

$y = n^{1/n}$

$\ln y = \frac{1}{n}\ln n$

$\lim_{n\to\infty} \frac{1}{n}\ln n$

$\lim_{n\to\infty} \frac{\ln n}{n} = \frac{\infty}{\infty}$

L'Hop

$\lim_{n\to\infty} \frac{1/n}{1} = \frac{1}{n} = 0$

$e^{\ln y} = e^0$

$y = e^0 = 1$

$\lim_{n\to\infty} 2\sqrt[n]{n}+1 = 2(1)+1 = 3 > 1$ The series diverges by the root test.

45) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

$\lim_{n\to\infty} \sqrt[n]{\frac{n}{3^n}}$

$\lim_{n\to\infty} \frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1$

The series converges by the root test

47) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2} \right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{1}{n} - \frac{1}{n^2}\right)^n}$

$\lim_{n\to\infty} \frac{1}{n} - \frac{1}{n^2} = 0 < 1$

The series converges by the root test

49) $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$

$\lim_{n\to\infty} \sqrt[n]{\frac{n}{(\ln n)^n}}$

$\lim_{n\to\infty} \frac{\sqrt[n]{n}}{\ln n} = \frac{1}{\infty} = 0 < 1$

The series converges by the root test