Calculus Section 9.6 Root Test

-Use the root test to determine convergence or divergence

Homework: page 633 #'s 35 – 49 odd

The final test to determine convergence or divergence is the root test. The root test is especially well suited to solve series involving n[th] powers.

Root Test

Let $\sum a_n$ be a series.

1) $\sum a_n$ converges absolutely if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$
2) $\sum a_n$ diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$
3) The Root Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$

Example) Using the Root Test

1) $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

$\lim_{n\to\infty} \sqrt[n]{\frac{e^{2n}}{n^n}}$

$\lim_{n\to\infty} \frac{e^2}{n} = 0 < 1$

The series converges by the root test.

2) $\sum_{n=1}^{\infty} \left(\frac{-3n}{2n+1}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{3n}{2n+1}\right)^n}$

$\lim_{n\to\infty} \frac{3n}{2n+1} = \frac{3}{2} > 1$

The series diverges by the root test.

3) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

$\lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}}$

$\lim_{n\to\infty} \frac{\sqrt[n]{n}}{2}$

$y = \sqrt[n]{n} = n^{1/n}$

$\ln y = \frac{1}{n} \ln n$

$\lim_{n\to\infty} \frac{1}{n} \ln n = \frac{\ln n}{n} = \frac{\infty}{\infty}$

l'Hop

$\lim_{n\to\infty} \frac{1/n}{1} = \frac{1}{n} = 0$

$\ln y = 0 \rightarrow e^{\ln y} = e^0 \rightarrow y = 1$

$\lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1$

The series converges by the root test.

4) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

$\lim_{n\to\infty} \sqrt[n]{\frac{(n!)^n}{n^{2n}}}$

$\lim_{n\to\infty} \frac{n!}{n^2} = \infty > 1$

The series diverges by the root test.